Elementary Analysis Math 140B—Winter 2007
Homework answers—Assignment 16; February 27, 2007

The pointwise limit of a sequence of monotonic functions (some non-increasing, some non-decreasing) is monotonic.

Solution: Let the sequence be denoted by (f_n) and let f denote the pointwise limit function. Suppose that infinitely many functions in the sequence are nondecreasing. These non-decreasing functions form a subsequence (f_{n_k}) of the sequence (f_n). If $x < y$ then $f_{n_k}(x) \geq f_{n_k}(y)$, so passing to the limit, $f(x) \geq f(y)$.

Now suppose that only finitely many of the functions in the sequence are nondecreasing. Then infinitely many must be nonincreasing. These nonincreasing functions form a subsequence (f_{n_k}) of the sequence (f_n). If $x < y$ then $f_{n_k}(x) \leq f_{n_k}(y)$, so passing to the limit, $f(x) \leq f(y)$. □