(a) Let \( f \) be a pure jump function with jumps \( j_n = 1/2^n \) at \( x_n = 1/3^n \). Show that \( f(0) = 0 \). Then assuming that the right hand derivative of \( f \) at zero, namely

\[
f'_+(0) := \lim_{h \to 0^+} \frac{f(h) - f(0)}{h}
\]

exists, show that it is equal to \( +\infty \).

**Solution:** Recall that a pure jump function is a function of the form \( f = \sum_{n=1}^{\infty} f_n \), where \( f_n(x) = j_n \) for \( x \geq x_n \) and \( f_n(x) = 0 \) for \( x < x_n \). Here, \((x_n)\) is a sequence of real numbers and \((j_n)\) is a sequence of positive numbers with \( \sum_j j_n < \infty \).

It is obvious that \( f(0) = 0 \) since \( f_n(0) = 0 \) for all \( n \geq 1 \). Now, taking a sequence \( h_m \to 0^+ \), where \( h_m \in (3^{-(m+1)}, 3^{-m}) \), you have \( f(h_m) = \sum_{k=m+1}^{\infty} 2^{-k} = 2^{-m} \) so that \( f(h_m)/h_m = 2^{-m}/h_m > 2^{-m}3^m = (3/2)^m \to +\infty \).

(b) Let \( g \) be a pure jump function with jumps \( j_n = 1/3^n \) at \( x_n = 1/2^n \). Show that \( g(0) = 0 \). Then assuming that the right hand derivative of \( g \) at zero, namely

\[
g'_+(0) := \lim_{h \to 0^+} \frac{g(h) - g(0)}{h}
\]

exists, show that it is equal to 0.

**Solution:** As in part (a), it is obvious that \( g(0) = 0 \). Now, taking a sequence \( h_m \to 0^+ \), where \( h_m \in (2^{-(m+1)}, 2^{-m}) \), you have \( g(h_m) = \sum_{k=m+1}^{\infty} 3^{-k} = 3^{-m} \) so that \( g(h_m)/h_m = 3^{-m}/h_m < 3^{-m}2^m = (2/3)^m \to 0 \).