Exercise 32.7

Let $f$ be integrable on $[a, b]$ and suppose that $g$ is a function on $[a, b]$ such that $g(x) = f(x)$ except for finitely many $x$ in $[a, b]$. Show that $g$ is integrable and that $\int_a^b f = \int_a^b g$.

Solution: See the text, Ross, page 336.

Exercise 33.9

Let $(f_n)$ be a sequence of integrable functions on $[a, b]$, and suppose that $f_n \to f$ uniformly on $[a, b]$. Prove that $f$ is integrable and that $\int_a^b f = \lim_{n \to \infty} \int_a^b f_n$. Compare this result with Theorem 25.2.

Solution: See the text, Ross, page 336.

Exercise 33.11

Let $f(x) = x \text{ sgn}(\sin \frac{1}{x})$ for $x \neq 0$ and $f(0) = 0$.

(a) Show that $f$ is not piecewise continuous on $[-1, 1]$.

Solution: See the text, Ross, page 336.

(b) Show that $f$ is not piecewise monotonic on $[-1, 1]$.

Solution: See the text, Ross, page 336.

(c) Show that $f$ is integrable on $[-1, 1]$.

Solution: See the text, Ross, page 336-337.