Elementary Analysis Math 140B—Winter 2007
Homework answers—Assignment 6; January 29, 2007

Exercise 25.2, page 190

Let

$$(f_n) = \frac{x^n}{n}.$$

Show that (f_n) is uniformly convergent on $[-1, 1]$ and specify the limit function.

Solution: If $x \in [-1, 1]$, then

$$|f_n(x)| \leq 1/n$$

so $f_n \to 0$ uniformly on $[-1, 1]$ by the first domination principle (see the minutes of January 12)

Exercise 25.6, page 191

(a) Show that if $\sum |a_k| < \infty$, then $\sum a_k x^k$ converges uniformly on $[-1, 1]$ to a continuous function.

Solution: If $x \in [-1, 1]$, then $|a_k x^k| \leq |a_k|$ for $k \geq 1$, so by the Weierstrass M-test (which I called the second domination principle in the minutes of January 12), the convergence of the series $\sum a_k x^k$ is uniform on $[-1, 1]$. The partial sums are polynomials, and are therefore continuous functions. The series $\sum a_k x^k$ is the uniform limit of the partial sums so it represents a continuous function, by Theorem 24.3.

(b) Does $\sum_{n=1}^{\infty} \frac{1}{n^2} x^n$ represent a continuous function on $[-1, 1]$?

Solution: YES. By (a), since $\sum \frac{1}{n^2} < \infty$.