Exercise 25.2, page 190

Let 

\[(f_n) = \frac{x^n}{n}.\]

Show that \((f_n)\) is uniformly convergent on \([-1, 1]\) and specify the limit function.

**Solution:** If \(x \in [-1, 1]\), then

\[|f_n(x)| \leq 1/n\]

so \(f_n \to 0\) uniformly on \([-1, 1]\) by the first domination principle (see the minutes of January 12).

Exercise 25.6, page 191

(a) Show that if \(\sum |a_k| < \infty\), then \(\sum a_kx^k\) converges uniformly on \([-1, 1]\) to a continuous function.

**Solution:** If \(x \in [-1, 1]\), then \(|a_kx^k| \leq |a_k|\) for \(k \geq 1\), so by the Weierstrass \(M\)-test (which I called the second domination principle in the minutes of January 12), the convergence of the series \(\sum a_kx^k\) is uniform on \([-1, 1]\). The partial sums are polynomials, and are therefore continuous functions. The series \(\sum a_kx^k\) is the uniform limit of the partial sums so it represents a continuous function, by Theorem 24.3.

(b) Does \(\sum_{n=1}^{\infty} \frac{1}{n^2}x^n\) represent a continuous function on \([-1, 1]\)?

**Solution:** YES. By (a), since \(\sum \frac{1}{n^2} < \infty\).