FORUM

- First Question April 19

  In question 4 of HW 1, I did this problem by expanding both sides into a long and nasty sum of products of A, B, X, and Y, and show that the results are the same on both sides. I wonder if there is a more 'elegant' way for this question.

- Answer to First Question April 19

  There is a more elegant approach, and it is contained in Exercise 7. By exercise 6, $M_n(C)$ is a Jordan algebra under circle multiplication. Therefore Exercise 4 is a special case of Exercise 7.

- Second Question April 19

  In question 7 of HW1, in part i), I did the replacement and expanded both sides.

  For the LHS, I get: $2u((uw)v) + u(w^2v) + w(u^2v) + 2w((uw)v)$

  For the RHS, I get: $2(uw)(uv) + w^2(uv) + u^2(wv) + 2(uw)(wv)$

  I do not know how to get rid of the two extra terms on both sides.

- Answer to Second Question April 19

  In the equation LHS=RHS, replace w by cw, where c is a nonzero number. Divide by c in the resulting equation. Then take the limit as c approaches 0.

- Question April 21

  What is an example of an algebra which does not have a unit element. (A unit element, or identity element, in an algebra A is an element e such that $ex = xe = x$ for every $x \in A$.

- Answer to Question April 21

  See the scan on the next page

(scan) 199c0423211.pdf

Example 1 Any Lie algebra $A \neq \{0\}$ (If e was an identity element, then $e^2 = 0 = e.e = e$, so $A = \{0\}$)

Example 2 The set of all sequences of real numbers which converge to 0, with addition $(x_i) + (y_i) = (x_i + y_i)$, multiplication $(x_i)(y_i) = (x_i y_i)$.

Example 3 The set M of all infinite matrices which have only finitely many non-zero entries:

$$M_1 = \left\{ \begin{bmatrix} a_{11} & 0 & 0 & \cdots \\ 0 & 0 & \cdots & \\ 0 & & & \\ \vdots & & \bigcirc & \end{bmatrix} : a_{11} \in \mathbb{R} \right\} \qquad M_2 = \left\{ \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & \cdots & 0 \\ a_{21} & a_{22} & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & & & \\ 0 & & & & & \\ 0 & & & & & \\ \vdots & & & & & \end{bmatrix} \; a_{ij} \in \mathbb{R} \right\}$$

let $M = M_1 \cup M_2 \cup \cdots$

If $a, b \in M$ say $a \in M_n$ $b \in M_m$

then a, b both belong to $M_{\max(m,n)} \subset M$

so can be multiplied and $ab \in M_{\max(m,n)} \subset M$

So M is an algebra

Example 4 The set of all polynomials which vanish at 0. (If $I(x) = \sum_{k=1}^{n} a_k x^k$ were an identity element then $If = f$ for $f(x) = \sum_{k=1}^{m} b_k x^k$ implies that $b_1 = b_2 = \cdots = b_m = 0$)

Example 5 *Proposition* An evolution algebra has an identity element if and only if $\exists$ natural basis $e_i^2 = k_i x_i$ $k_i \neq 0$

- Question May 3

  When I was working on problem 2 of homework 2, I managed to do parts 1-3 of part (b), but I got stuck on part 4. I tried to prove this by contradiction, but I do not know how to use the properties of power associative. Could you give me some hints?

- Answer to Question May 3

  Let $M_B(A) = [w_{ij}], B = \{e_1 \ldots, e_n\}$

  (1) From April 21 pdffile, for any $i = 1, \ldots n$,

$$e_i^2(e_ie_i) = \sum_{k=1}^{n} w_{ki}e_k^2, \quad (e_i^2e_i) = w_{ii}e_i^2$$

  (2) From (a) and (b) of Exercise 2(b),

$$e_1^2, \ldots, e_n^2 \text{ is a basis.}$$