1990042421 _ 6 PP

Roberts 1975

4/23/21

§2 digraph = directed graph D : Vertices $V = \{x, y, \ldots\}$

arcs $A \subset V \times V$ (x,y), loops (x,x)

signed digraph = each arc has + or −

sequence = a set of vertices $x_1, \ldots, x_t$, (x_i, x_{i+1}) is an arc

sign of a sequence = product of signs of its arcs

length of a sequence = the number of arcs

cycle = arc $\underbrace{x_1, x_2, \ldots, x_{t-1}}_{\text{distinct}}, x_t = x_1$

[9] F.S. Roberts "signed digraphs and ...." pp 395-410

vertices represent variables $\boxed{\text{(no citations) (POSTED) (PRO) Roberts[9]1971.pdf}}$

arc (x,y) means a change in x affects a change in y

$+ \text{ if } \begin{cases} x\uparrow \Rightarrow y\uparrow \\ x\downarrow \Rightarrow y\downarrow \end{cases}$ $\quad - \text{ if } \begin{matrix} x\uparrow \Rightarrow y\downarrow \\ x\downarrow \Rightarrow y\uparrow \end{matrix}$

example : Figure 1 on p. 578

change of sign p. 578

cycles, C, F, U, C , C, R, U, C , augmenting, unstable p. 579

[2] Book "Structural Models" An introduction ...

Langson H61 H287 (HathiTrust online)

(Harary, Norman, Cartwright. 1965)

SKIP §3 for today 4/24/21

§3 weighted digraph = each arc has a weight $w(x,y) \in \mathbb{R}$
(disregard time lags)

Figure 2 p. 580

[10] Roberts RAND report R-1578-NSF
(PRO) Signed-digraph.pdf

(PRO) PULSE120720 pt2 _ 6PP. pdf
(PRO) PULSE120520pt1 _ 5PP. pdf

§4 signed digraph : vertices $x_1, x_2, \ldots, x_n$

$v_i(t)$ = the value at x_i at time $t = 0, 1, 2, \ldots$

$p_i^o(t+1)$ = external pulse introduced at x_i at time $t+1$
($p_i^o(0)$ also has a value)

(1) $$v_i(t+1) = v_i(t) + p_i^o(t+1) + \sum_j \operatorname{sgn}(x_j, x_i)\, p_j(t)$$

$$\operatorname{sgn}(x_j, x_i) = \begin{cases} +1 & \text{if } (x_j, x_i) \text{ is } + \\ -1 & \text{if } (x_j, x_i) \text{ is } - \\ 0 & \text{if } (x_j, x_i) \text{ is not an arc} \end{cases}$$

$p_j(t)$ = the pulse at vertex x_i at time t

$$= \begin{cases} v_j(t) - v_j(t-1) & \text{if } t > 0 \\ p_j^o(0) & \text{if } t = 0 \end{cases}$$

DEF A pulse process on a signed digraph D is defined by (1), an initial vector of initial values $V(0) = (v_1(0), \ldots, v_n(0))$

by external pulse vectors $P^o(t) = (p_1^o(t), \ldots, p_n^o(t))$ at time t.

and by the pulse vector $P(t) = (p_1(t), \ldots, p_n(t))$

Figure 3. p. 582

pulse process on a weighted digraph

(2) $$v_i(t+1) = v_i(t) + p_i^{\circ}(t+1) + \sum_j w(x_j, x_i)\, p_j(t)$$

(system of finite difference equations :)

$$p_i(t+1) = p_i^{\circ}(t+1) + \sum_j w(x_j, x_i)\, p_j(t)$$

autonomous pulse process $=$ $P^{\circ}(t) = 0 \quad \forall t > 0$

simple pulse process $=$ autonomous, $P^{\circ}(0) = (0, .., 1, 0 ... 0)$
starting at vertex x_i i^{th} place

SKIP THIS THEOREM for TODAY 4/24/21

Theorem 1 In a simple pulse process starting at vertex x_i of a signed digraph D, $p_j(t)$ is given by the signed number of sequences from x_i to x_j of length t and $v_j(t)$ is given by $v_j(\text{start}) + p_j^{\circ}(0) +$ the signed number of sequences from x_i to x_j of length at most t.

$$P^{\circ}(0) = (p_1^{\circ}(0), \ldots, p_n^{\circ}(0))$$

$$= (0, \ldots, \underset{\underset{1}{\shortparallel}}{p_i^{\circ}(0)}, 0 \ldots 0)$$

$$P(t) = V(t) - V(t-1) \quad t > 0$$

$$P(0) = P^{\circ}(0) \quad t = 0$$

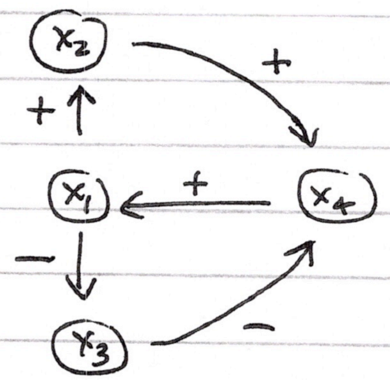
$$V(0) = (v_1(0), \ldots, v_n(0))$$

$$v_i(0) = v_i(\text{start}) + p_i^{\circ}(0)$$

$$V(\text{start}) = (v_1(\text{start}), \ldots, v_n(\text{start}))$$

$$V(0) = V(\text{start}) + P^{\circ}(0)$$

Figure 3



assumptions

$$P^{\circ}(0) = (1, 0, 0, 0)$$

$$p^{\circ}(t) = 0 \quad t > 0$$

(simple pulse process starting at x_1)

$$V(\text{start}) = (0, 0, 0, 0)$$

$$V(0) = (1, 0, 0, 0) \qquad P(0) = P^{\circ}(0) = (1, 0, 0, 0)$$

$$(p_1(0), p_2(0), p_3(0), p_4(0))$$

$$V(1) = (1, 1, -1, 0)$$

$$P(1) = V(1) - V(0) = (0, 1, -1, 0)$$

$$V(2) = (1, 1, -1, 1+1) = (1, 1, -1, 2)$$

$$P(2) = V(2) - V(1) = (0, 0, 0, 2)$$

$$V(3) = (3, 1, -1, 2)$$

autonomous

$$v_i(t+1) = v_i(t) + 0 + \sum_{j=1}^{4} w(x_j, x_i)\,\underbrace{(v_j(t) - v_j(t-1))}_{p_j(t) \text{ if } t>0}$$

$(p_j(0) = p_j^{\circ}(0))$

$$V(t) = (v_1(t), v_2(t), v_3(t), v_4(t))$$

$$V(0) = (1, 0, 0, 0)$$

$t=0$ to $t=1$

$$v_1(1) = v_1(0+1) = v_1(0) + 0 + w(x_1,x_1)p_1(0) + w(x_2,x_1)p_2(0) + w(x_3,x_1)p_3(0) + w(x_4,x_1)p_4(0)$$

$$= 1 + 0 + 0\cdot p_1(0) + 0\cdot p_2(0) + 0\cdot p_3(0) + 1\cdot \underbrace{p_4(0)}_{0}$$

$$= \boxed{1}$$

$$v_2(1) = v_2(0+1) = \underbrace{v_2(0)}_{0} + 0 + \underbrace{w(x_1,x_2)}_{+1}\,\underbrace{p_1(0)}_{1}$$ (nothing else goes into x_2)

$$= \boxed{1}$$

$$v_3(1) = \underbrace{v_3(0)}_{0} + \underbrace{w(x_1,x_3)}_{-1}\,\underbrace{p_1(0)}_{1} = \boxed{-1}$$

$$v_4(1) = \underbrace{v_4(0)}_{0} + \underbrace{w(x_2,x_4)}_{+1}\,\underbrace{p_2(0)}_{0} + \underbrace{w(x_3,x_4)}_{-1}\cdot\underbrace{p_3(0)}_{0} = \boxed{0}$$

$$V(1) = (v_1(1), \ldots, v_4(1)) = (1, 1, -1, 0);\quad P(1) = V(1) - V(0) = (0, 1, -1, 0)$$
$$= (p_1(1), p_2(1), p_3(1), p_4(1))$$

$t=1$ to $t=2$

$$v_1(2) = v_1(1+1) = v_1(1) + 0 + w(x_4,x_1)p_4(1)$$
$$= 1 + 1\cdot 0 = \boxed{1}$$

$$v_2(2) = v_2(1) + w(x_1,x_2)p_1(1)$$
$$= 1 + 1\cdot 0 = \boxed{1}$$

$$v_3(2) = v_3(1) + w(x_1,x_3)p_1(1)$$
$$= -1 + (-1)\cdot 0 = \boxed{-1}$$

$$v_4(2) = v_4(1) + w(x_3,x_4)p_3(1) + w(x_2,x_4)p_2(1)$$
$$0 + (-1)(-1) + 1\cdot 1 = \boxed{2}$$

$t=2$ to $t=3$

$V(2) = (1,1,-1,2) \quad V(1) = (1,1,-1,0)$

$P(2) = V(2) - V(1) = (0,0,0,2)$

$v_1(3) = v_1(2) + w(x_4,x_1)\, p_4(2)$

$1 + 1 \cdot 2 = \boxed{3}$

$v_2(3) = v_2(2) + w(x_1,x_2)\, p_1(2)$

$1 + 1 \cdot 0 = \boxed{1}$

$v_3(3) = v_3(2) + w(x_1,x_3)\, p_1(2)$

$-1 + (-1) \cdot 0 = \boxed{-1}$

$v_4(3) = v_4(2) + w(x_2,x_4)\, p_2(2) + w(x_3,x_4)\, p_3(2)$

$= 2 + 1 \cdot 0 + (-1) \cdot 0 = \boxed{2}$

$V(3) = (3,1,-1,2) \quad V(2) = (1,1,-1,2)$

$P(3) = V(3) - V(2) = (2,0,0,0)$