Complex Analysis Math 147—Winter 2006
Assignment 12
due March 14, 2008

1. Let f be an entire function and suppose $|f(z)| \leq M|z|$ for every $z \in \mathbf{C}$. Show that $f(z) = cz$ for some constant $c \in \mathbf{C}$ and all $z \in \mathbf{C}$.

2. Show that there is no power series $f(z) = \sum_0^\infty c_n z^n$ such that

   (i) $f(z) = 1$ for $z = 1/2, 1/3, \ldots, 1/n, \ldots$

   (ii) $f'(0) > 0$

3. Show that if $f : \mathbf{C} \to \mathbf{C}$ is a continuous function such that f is known to be analytic on the complement of the interval $[-1, 1]$, then in fact f is an entire function.

4. Show that if f and g are analytic on a domain D and if $\overline{f}g$ is also analytic, then either f is a constant, or g is identically zero.