Complex Analysis Math 147—Winter 2006
Assignment 5
due February 17, 2006

Part 1

1. Determine the domain of analyticity for each of the given functions and explain why
$$\int_C f(z)\,dz = 0,$$
where C is the circle of radius 2 centered at 0, traversed in the counter clockwise direction once.
   (a) $f(z) = \frac{z}{z^2+25}$
   (b) $f(z) = e^{-z}(2z+1)$
   (c) $f(z) = \frac{\sin z}{z^2-6z+10}$

2. Given that D is a domain containing the closed curve C, that z_0 is a point not in D, and that $\int (z-z_0)^{-1}\,dz \neq 0$, explain why D is not simply connected.

3. Evaluate $\int_{C_1} 1/(z^2+1)\,dz$ and $\int_{C_2} 1/(z^2+1)\,dz$ where C_1 is the circle of radius $1/2$ and center i traversed in the counter clockwise direction once, and C_2 is the square with center 0 and perimeter 16, traversed in the clockwise direction once.

4. Evaluate $\int_C \frac{z}{(z+2)(z-1)}\,dz$, where C is the circle with radius 4 and center 0 traversed in the counter-clockwise direction once. What about if C is traversed in the clockwise direction? What if C is traversed in the clockwise direction twice?

5. Suppose that f is of the form
$$f(z) = \frac{A_k}{z^k} + \frac{A_{k-1}}{z^{k-1}} + \cdots + \frac{A_1}{z} + g(z)$$
where $k \geq 1$, g is analytic inside and on the unit circle $|z| = 1$, and $A_1, \ldots, A_k$ are complex numbers. If C is the unit circle traversed in the counter-clockwise direction once, calculate $\int_C f(z)\,dz$.

6. For $R > 2$, set $I(R) = \int_{C_R} \frac{dz}{z^2(z-1)^3}$, where C_R is the circle of radius R and center 0 traversed in the clockwise direction once.
   (a) Show that $|I(R)| \leq \frac{2\pi}{R(R-1)^3}$
   (b) Show that $\int_C \frac{dz}{z^2(z-1)^3} = 0$, where C is the circle of radius 2 and center 0 traversed in the clockwise direction once.

Part 2

Exercises 1–4 of chapter 6 of Cain.