Complex Analysis Math 220C—Spring 2008
Assignments 1–6

Assignment 1 Problems 14 and 15 of chapter 6 of the text. These will de discussed in the informal discussion section on Friday April 4 at 4 pm in MSTB 256. The due date for this assignment is Friday April 11.

Assignment 2 Exercises 9,10,11,12 of chapter 10 of Greene-Krantz. These will be discussed in the discussion section on Friday April 11 at 4 pm in MSTB 256. The due date for this assignment is Friday April 18.

Assignment 3 (Due April 25)

Suppose f is analytic in $B(a,R)-\{a\}$ except for a sequence of poles $\{z_1,z_2,\ldots\}$ which converges to a. (Note that although f is not assumed to be analytic at a, nevertheless, a is not an isolated singularity of f.) Show that $f(B(a,R)-\{a,z_1,\ldots\})$ is dense in the complex plane.

> Hint: If it is not true, consider $g(z)=1/(f(z)-w)$ for $z\in G:=B(a,R)-\{a,z_1,z_2,\ldots\}$, where $w\in\mathbf{C}$ and $\delta>0$ are such that $|f(z)-w|>\delta$ for all $z\in G$. Then obtain a contradiction by proving the following statements.

(a) There is an analytic extension h of g to $B(a,R)-\{a\}$ which vanishes at each z_n.

(b) The point a is an isolated singularity of h which is a removable singularity of h, and the analytic extension of h to a vanishes at a.

Assignment 4 (Due April 25) Problem 1 and one (your choice) of Problems 2,3,4 in Chapter 10 of Greene-Krantz.

Assignment 5 Due April 25

For a fixed complex number a with $|a|<1$, define a function φ_a by

$$\varphi_a(z)=\frac{z+a}{1+\overline{a}z}.$$

Although $\varphi_a(z)$ is defined for all $z\neq -1/\overline{a}$, we shall consider it as a function on the closed unit disk $|z|\leq 1$. Prove the following statements.

(a) *If $|z|<1$ then $|\varphi_a(z)|<1$.*

(b) *If $|z|=1$ then $|\varphi_a(z)|=1$.*

(c) *φ_a is a one to one function, that is, if $|z_1|<1,|z_2|<1$ and if $\varphi_a(z_1)=\varphi_a(z_2)$, then $z_1=z_2$.*

(d) *φ_a is an onto function, that is, if $|w_0|<1$, then there is a z_0 with $|z_0|<1$ and $\varphi_a(z_0)=w_0$.*

(e) *What is the inverse of φ_a?*

Assignment 6 Due April 25

Let f be an arbitrary analytic function on the unit disk $|z|<1$ which is one to one and onto, that is, if $|z_1|<1,|z_2|<1$ and if $f(z_1)=f(z_2)$, then $z_1=z_2$; and if $|w_0|<1$, then there is a z_0 with $|z_0|<1$ and $f(z_0)=w_0$. Prove the following statements.

(a) *If $f(0) = 0$, then $f(z) = e^{i\theta} z$ for some real θ.*

(b) *If $f(0) = a \neq 0$, let $g(z)$ be defined by $g(z) = \varphi_{-a}(f(z))$. Then $g(z) = e^{i\theta} z$ for some real θ.*

(c) *The function f has the form*

$$f(z) = e^{i\theta} \varphi_a(z),$$

for some θ real and $|a| < 1$.

Assignment 7 Due on May 16. Complete the proof of Picard's little theorem (Case 3).

Assignment 8 Due May 12. You should review all the problems below and write up any two of them from each page (total of 10 problems)

Conway's book

page 43: #9,11,12,13,14,15,16,17,18,19,20,21 (ANY TWO)
page 54: #8,9,18,19,24 (ANY TWO)
page 80:#1,4,5,6,7,8,9,10 (ANY TWO)
page 129:#1,2,3,4,5,6,7,8 (ANY TWO)
page 132:#1,2,3,4,5,6,7,8 (ANY TWO)