Assignment 1 Problems 14 and 15 of chapter 6 of the text. These will be discussed in the informal discussion section on Friday April 4 at 4 pm in MSTB 256. The due date for this assignment is Friday April 11.

Assignment 2 Exercises 9,10,11,12 of chapter 10 of Greene-Krantz. These will be discussed in the discussion section on Friday April 11 at 4 pm in MSTB 256. The due date for this assignment is Friday April 18.

Assignment 3 (Due April 25)
Suppose $f$ is analytic in $B(a, R) - \{a\}$ except for a sequence of poles $\{z_1, z_2, \ldots\}$ which converges to $a$. (Note that although $f$ is not assumed to be analytic at $a$, nevertheless, $a$ is not an isolated singularity of $f$.) Show that $f(B(a, R) - \{a, z_1, \ldots\})$ is dense in the complex plane.

Hint: If it is not true, consider $g(z) = 1/(f(z) - w)$ for $z \in G := B(a, R) - \{a, z_1, z_2, \ldots\}$, where $w \in \mathbb{C}$ and $\delta > 0$ are such that $|f(z) - w| > \delta$ for all $z \in G$. Then obtain a contradiction by proving the following statements.

(a) There is an analytic extension $h$ of $g$ to $B(a, R) - \{a\}$ which vanishes at each $z_n$.
(b) The point $a$ is an isolated singularity of $h$ which is a removable singularity of $h$, and the analytic extension of $h$ to $a$ vanishes at $a$.

Assignment 4 (Due April 25) Problem 1 and one (your choice) of Problems 2,3,4 in Chapter 10 of Greene-Krantz.

Assignment 5 Due April 25
For a fixed complex number $a$ with $|a| < 1$, define a function $\varphi_a$ by

$$\varphi_a(z) = \frac{z + a}{1 + \overline{a}z}.$$ 

Although $\varphi_a(z)$ is defined for all $z \neq -1/\overline{a}$, we shall consider it as a function on the closed unit disk $|z| \leq 1$. Prove the following statements.

(a) If $|z| < 1$ then $|\varphi_a(z)| < 1$.
(b) If $|z| = 1$ then $|\varphi_a(z)| = 1$.
(c) $\varphi_a$ is a one to one function, that is, if $|z_1| < 1, |z_2| < 1$ and if $\varphi_a(z_1) = \varphi_a(z_2)$, then $z_1 = z_2$.
(d) $\varphi_a$ is an onto function, that is, if $|w_0| < 1$, then there is a $z_0$ with $|z_0| < 1$ and $\varphi_a(z_0) = w_0$.
(e) What is the inverse of $\varphi_a$?

Assignment 6 Due April 25
Let $f$ be an arbitrary analytic function on the unit disk $|z| < 1$ which is one to one and onto, that is, if $|z_1| < 1, |z_2| < 1$ and if $f(z_1) = f(z_2)$, then $z_1 = z_2$; and if $|w_0| < 1$, then there is a $z_0$ with $|z_0| < 1$ and $f(z_0) = w_0$. Prove the following statements.
(a) If $f(0) = 0$, then $f(z) = e^{i\theta}z$ for some real $\theta$.

(b) If $f(0) = a \neq 0$, let $g(z)$ be defined by $g(z) = \varphi_{-a}(f(z))$. Then $g(z) = e^{i\theta}z$ for some real $\theta$.

(c) The function $f$ has the form

$$f(z) = e^{i\theta}\varphi_a(z),$$

for some $\theta$ real and $|a| < 1$.

Assignment 7 Due on May 16. Complete the proof of Picard’s little theorem (Case 3).

Assignment 8 Due May 12. You should review all the problems below and write up any two of them from each page (total of 10 problems)

Conway’s book

- page 43: #9,11,12,13,14,15,16,17,18,19,20,21 (ANY TWO)
- page 54: #8,9,18,19,24 (ANY TWO)
- page 80: #1,4,5,6,7,8,9,10 (ANY TWO)
- page 129: #1,2,3,4,5,6,7,8 (ANY TWO)
- page 132: #1,2,3,4,5,6,7,8 (ANY TWO)