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Global integration of Leibniz algebras. (English summary)

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Recall that a (real) Leibniz algebra is an $\mathbb{R}$-vector space $\mathfrak{h}$ equipped with a bilinear map $[-,-]\colon \mathfrak{h}\otimes\mathfrak{h}\to\mathfrak{h}$ satisfying the *Leibniz identity*:

$$[x,[y,z]]=[[x,y],z]-[[x,z],y],\quad x,y,z\in\mathfrak{h}.$$

A quintuple $(\mathfrak{h},\mathfrak{p},\mathfrak{g},[\cdot;\cdot]_{\mathfrak{g}},\dot\rho)$ is called a $\mathfrak{g}$-augmented Leibniz algebra if and only if the following holds:

1. $(\mathfrak{g};[\cdot;\cdot]_{\mathfrak{g}})$ is a (real) Lie algebra.
2. $\mathfrak{h}$ is an $\mathbb{R}$-vector space which is a left $\mathfrak{g}$-module via the $\mathbb{R}$-linear map $\dot\rho\colon g\otimes\mathfrak{h}\to\mathfrak{h}$ written $\dot\rho_\xi(x)=\xi.x$ for all $\xi\in\mathfrak{g}$ and $x\in\mathfrak{h}$.
3. $\mathfrak{p}\colon\mathfrak{h}\to\mathfrak{g}$ is an $\mathbb{R}$-linear morphism of $\mathfrak{g}$-modules, i.e., for all $\xi\in\mathfrak{g}$ and $x\in\mathfrak{h}$,

$$\mathfrak{p}(\xi.x)=[\xi,\mathfrak{p}(x)]_{\mathfrak{g}}.$$

Recall also that an augmented Lie rack [see R. A. Fenn and C. P. Rourke, J. Knot Theory Ramifications **1** (1992), no. 4, 343–406; MR1194995] is a quadruple (M,ϕ,G,l) consisting of a pointed differentiable manifold (M,e_M), of a Lie group G, of a smooth map $\phi\colon M\to G$ (of pointed manifolds), and of a smooth left G-action $l\colon G\times M\to M$ (written $(g;x)=l(g;x)=l_g(x)=gx$) such that for all $g\in G$, $x\in M$,

$$ge_M=e_M;\quad \phi(gx)=g\phi(x)g^{-1}.$$

One of the most challenging problems in the theory of Leibniz algebras is the generalization of Lie's "third theorem" to Leibniz algebras. Also known as the "coquecigrue problem", this problem was introduced by J.-L. Loday [Enseign. Math. (2) **39** (1993), no. 3-4, 269–293; MR1252069] in his attempt to quantify the lack of periodicity in algebraic K-theory, and it consists of finding a generalization of the notion of groups that integrates Leibniz algebras in such a way that in the case where the given Leibniz algebra is a Lie algebra, the associated "integral" is a Lie group. Several authors have published results related to this problem. In 2007, M. K. Kinyon [J. Lie Theory **17** (2007), no. 1, 99–114; MR2286884] showed that the tangent space of a Lie rack at some distinguished points has a Leibniz algebra structure, and that a solution to this integration problem for an arbitrary Leibniz algebra should include the notion of digroup as a special case. In 2013, S. Covez [Ann. Inst. Fourier (Grenoble) **63** (2013), no. 1, 1–35; MR3089194] provided a local solution to the "coquecigrue problem" by constructing a local Lie rack integrating a given (real, finite-dimensional) Leibniz algebra in such a way that in the case of Lie algebras, one obtains the conjugation Lie rack underlying the usual (simply connected) Lie group integrating it.

In this paper, the authors give a nonfunctorial construction of a (global) Lie rack that integrates a given (real, finite-dimensional) Leibniz algebra in such a way that in the case of a Lie algebra, the construction yields the conjugation rack underlying the usual (simply connected) Lie group integrating it. The main difference between this construction and Covez's construction is that instead of integrating a Leibniz algebra, the authors integrate an augmented Leibniz algebra into an augmented Lie rack without reducing the procedure to the integration of cocycles. *Guy Roger Biyogmam*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.