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The zygotic algebra is not an evolution algebra

zygotic algebra basis f_1, f_2, f_3

	f_1	f_2	f_3
f_1	f_1	$\frac{1}{2}(f_1+f_2)$	f_2
f_2	$\frac{1}{2}(f_1+f_2)$	$\frac{1}{2}f_1+\frac{1}{4}f_3+\frac{1}{2}f_2$	$\frac{1}{2}f_3+\frac{1}{2}f_2$
f_3	f_2	$\frac{1}{2}f_3+\frac{1}{2}f_2$	f_3

multiplication table

HW2 #1(b) show that M is the matrix with respect to $\{f_1,f_2,f_3\}$ of the linear transformation T defined by $Tf_i = e_i$ and that M is non-singular

$$M = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}$$

By way of contradiction, suppose $e_1\ e_2\ e_3$ is nat. basis

we shall get a contradiction.

$$e_i = \alpha_{1i} f_1 + \alpha_{2i} f_2 + \alpha_{3i} f_3 \qquad i = 1, 2, 3$$

$$\underset{i \neq j}{e_i e_j = 0} \Longrightarrow$$

$$(i,j) \in \{(1,2), (1,3), (2,3)\}$$

$$e_i e_j = (**) f_1 + (*) f_2 + (***) f_3 = 0$$

HW2 #1(a) Calculate the values of $(*)$, $(**)$, $(***)$

$$\left.\begin{aligned} &(**) \quad (2\alpha_{1i} + \alpha_{2i})(2\alpha_{1j} + \alpha_{2j}) = 0 \\ &(***) \quad (2\alpha_{3i} + \alpha_{2i})(2\alpha_{3j} + \alpha_{2j}) = 0 \\ &(*) \quad \alpha_{1i}\alpha_{2j} + 2\alpha_{1i}\alpha_{3j} + \alpha_{2i}\alpha_{1j} + \alpha_{2i}\alpha_{2j} + \alpha_{2i}\alpha_{3j} + 2\alpha_{3i}\alpha_{1j} + \alpha_{3i}\alpha_{2j} = 0 \end{aligned}\right\}$$

$(**)$ $2\alpha_{1i} + \alpha_{2i} = 0$ or $2\alpha_{1j} + \alpha_{2j} = 0$ (or both)

$(***)$ $2\alpha_{3i} + \alpha_{2i} = 0$ OR $2\alpha_{3j} + \alpha_{2j} = 0$ (or both)

$$(**)\begin{cases}(2\alpha_{11}+\alpha_{21})(2\alpha_{12}+\alpha_{22})=0 & (i,j)=(1,2)\\ (2\alpha_{11}+\alpha_{21})\ (2\alpha_{13}+\alpha_{23})=0 & (i,j)=(1,3)\\ (2\alpha_{12}+\alpha_{22})(2\alpha_{13}+\alpha_{23})=0 & (i,j)=(2,3)\end{cases}$$

What are all possibilities for the above ?

$$2\alpha_{11}+\alpha_{21}=0 \qquad\qquad 2\alpha_{12}+\alpha_{22}=0$$

$$2\alpha_{12}+\alpha_{22}=0 \quad 2\alpha_{13}+\alpha_{23}=0 \quad 2\alpha_{11}+\alpha_{21}=0 \quad 2\alpha_{13}+\alpha_{23}=0$$

(1) (2) (3) (4)

4 cases (**)

$$\left.\begin{array}{lll}(1) & 2\alpha_{11}+\alpha_{21}=0 & m=1\\ & 2\alpha_{12}+\alpha_{22}=0 & n=2\\ (2) & 2\alpha_{11}+\alpha_{21}=0 & m=1\\ & 2\alpha_{13}+\alpha_{23}=0 & n=3\\ (3) & 2\alpha_{12}+\alpha_{22}=0 & m=2\\ & 2\alpha_{11}+\alpha_{21}=0 & n=1\\ (4) & 2\alpha_{12}+\alpha_{22}=0 & m=2\\ & 2\alpha_{13}+\alpha_{23}=0 & n=3\end{array}\right\}$$

$$\exists\ m,n\in\{1,2,3\},\ m\neq n$$

$$2\alpha_{1m}+\alpha_{2m}=0$$
$$2\alpha_{1n}+\alpha_{2n}=0$$

$$(***)\begin{cases}(2\alpha_{31}+\alpha_{21})(2\alpha_{32}+\alpha_{22})=0 & (i,j)=(1,2)\\ (2\alpha_{31}+\alpha_{21})(2\alpha_{33}+\alpha_{23})=0 & (i,j)=(1,3)\\ (2\alpha_{32}+\alpha_{22})(2\alpha_{33}+\alpha_{23})=0 & (i,j)=(2,3)\end{cases}$$

similarly: 4 cases (***) $\Rightarrow \exists\ m,s\in\{1,2,3\}\quad m\neq s$

$$2\alpha_{3m}+\alpha_{2m}=0$$
$$2\alpha_{3s}+\alpha_{2s}=0$$

Thus $\exists\ m,n,s\in\{1,2,3\}\quad m\neq n,\ m\neq s$

$$\begin{aligned}2\alpha_{1m}+\alpha_{2m}&=0\\ 2\alpha_{1n}+\alpha_{2n}&=0\\ 2\alpha_{3m}+\alpha_{2m}&=0\\ 2\alpha_{3s}+\alpha_{2s}&=0\end{aligned}\quad\Rightarrow\quad\begin{aligned}\alpha_{1m}&=\alpha_{3m}\\ \alpha_{2m}&=-2\alpha_{1m}\\ \alpha_{2n}&=-2\alpha_{1n}\\ \alpha_{2s}&=-2\alpha_{3s}\end{aligned}$$

Two cases $n\neq s$ or $n=s$

Define vectors
$$v_m=\alpha_{1m}e_1\overbrace{-2\alpha_{1m}}^{\alpha_{2m}}e_2+\alpha_{3m}e_3$$
$$v_n=\alpha_{1n}e_1\overbrace{-2\alpha_{1n}}^{\alpha_{2n}}e_2+\alpha_{3n}e_3$$
$$v_s=\alpha_{1s}e_1\overbrace{-2\alpha_{3s}}^{\alpha_{2s}}e_2+\alpha_{3s}e_3$$

so v_m, v_n, v_s are linearly independent if $n\neq s$

$\boxed{\text{CASE 1}\quad n\neq s}$

$i=m,\ j=s$ in $(*)$:

$$\alpha_{1n}\alpha_{2s}+2\alpha_{1n}\alpha_{3s}+\alpha_{2n}\alpha_{1s}+\alpha_{2n}\alpha_{2s}+\alpha_{2n}\alpha_{3s}+2\alpha_{3n}\alpha_{1s}+\alpha_{3n}\alpha_{2s}=0$$

$$\cancel{\alpha_{1n}(-2\alpha_{3s})}+\cancel{2\alpha_{1n}\alpha_{3s}}-2\alpha_{1n}\alpha_{1s}-2\alpha_{1n}(-2\alpha_{3s})-2\alpha_{1n}\alpha_{3s}+2\alpha_{3n}\alpha_{1s}+\alpha_{3n}(-2\alpha_{3s})=0$$

$$-2\alpha_{1n}\alpha_{1s}+2\alpha_{1n}\alpha_{3s}+2\alpha_{3n}\alpha_{1s}-2\alpha_{3n}\alpha_{3s}=0$$

$$(\alpha_{1n}-\alpha_{3n})(\alpha_{3s}-\alpha_{1s})=\alpha_{1n}\alpha_{3s}-\alpha_{3n}\alpha_{3s}-\alpha_{1n}\alpha_{1s}+\alpha_{3n}\alpha_{1s}=0$$

So either $\alpha_{1n} = \alpha_{3n}$ OR $\alpha_{3s} = \alpha_{1s}$

$$v_m = \alpha_{1m}(e_1 - 2e_2 + e_3)$$
$$v_n = \alpha_{1n}(e_1 - 2e_2 + e_3)$$
$$v_s = \alpha_{1s} - 2\alpha_{3s} e_2 + \alpha_{3s} e_3$$

contradiction, not linearly independent (v_m, v_n) not LI

$$v_m = \alpha_{1m}(e_1 - 2e_2 + e_3)$$
$$v_n = \alpha_{1n} e_1 - 2\alpha_{1n} e_2 + \alpha_{3n} e_3$$
$$v_s = \alpha_{1s}(e_1 - 2e_2 + e_3)$$

(v_m, v_s) not LI

case 2 $n = s$

Then $\alpha_{1m} = \alpha_{3m}$

$\alpha_{1n} = \alpha_{3n}$

$\alpha_{2m} = -2\alpha_{1m}$

$\alpha_{2n} = -2\alpha_{1n}$

So

$$v_m = \overset{\alpha_{3m}}{\alpha_{1m}} e_1 - \overset{-2\alpha_{3m}}{2\alpha_{1m}} e_2 + \alpha_{3m} e_3 = \alpha_{3m}(e_1 - 2e_2 + e_3)$$

$$v_5 = v_n = \overset{\alpha_{3n}}{\alpha_{1n}} e_1 - \overset{-2\alpha_{3n}}{2\alpha_{1n}} e_2 + \alpha_{3n} e_3 = \alpha_{3n}(e_1 - 2e_2 + e_3)$$

contradiction, v_m, v_n not linearly independent. $\blacksquare$