

4/13/21

The zygotic algebra is not an evolution algebra

zygotic algebra basis f_1, f_2, f_3

	f_1	f_2	f_3
f_1	f_1	$\frac{1}{2}(f_1+f_2)$	f_2
f_2	$\frac{1}{2}(f_1+f_2)$	$\frac{1}{2}f_1 + \frac{1}{4}f_3 + \frac{1}{2}f_2$	$\frac{1}{2}f_3 + \frac{1}{2}f_2$
f_3	f_2	$\frac{1}{2}f_3 + \frac{1}{2}f_2$	f_3

multiplication table

HW2 # 1(b)

show that M is the matrix with respect to $\{f_1, f_2, f_3\}$ of the linear transformation T defined by $Tf_i = e_i$ and that M is non-singular

$$M = \begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix}$$

By way of contradiction,

Suppose e_1, e_2, e_3 is nat. basis we shall get a contradiction.

$$e_i = d_{1i}f_1 + d_{2i}f_2 + d_{3i}f_3 \quad i=1,2,3$$

$$e_i e_j = 0 \Rightarrow i \neq j$$

$$e_i e_j = (**)f_1 + (*)f_2 + (***)f_3 = 0$$

$(i,j) \in \{(1,2), (1,3), (2,3)\}$

HW2 # 1(a) Calculate the values of $(**), (***), (***)$

$$(**) \quad (2d_{1i} + d_{2i})(2d_{1j} + d_{2j}) = 0$$

$$(***) \quad (2d_{3i} + d_{2i})(2d_{3j} + d_{2j}) = 0$$

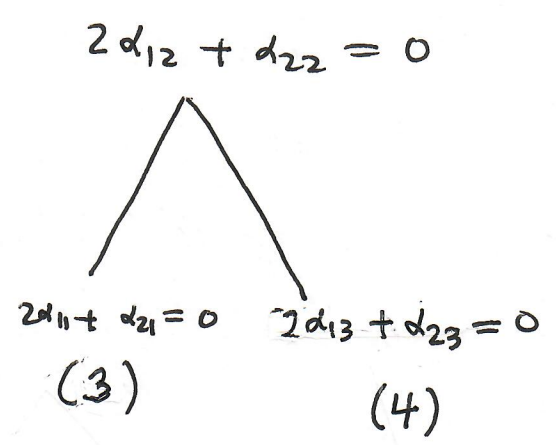
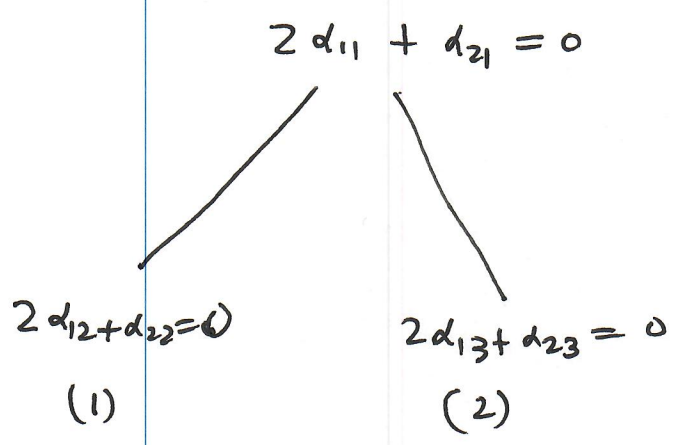
$$(*) \quad d_{1i}d_{2j} + 2d_{1i}d_{3j} + d_{2i}d_{1j} + d_{2i}d_{2j} + d_{2i}d_{3j} + 2d_{3i}d_{1j} + d_{3i}d_{2j} = 0$$

$$(**) \quad 2d_{1i} + d_{2i} = 0 \quad \text{or} \quad 2d_{1j} + d_{2j} = 0 \quad (\text{or both})$$

$$(***) \quad 2d_{3i} + d_{2i} = 0 \quad \text{OR} \quad 2d_{3j} + d_{2j} = 0 \quad (\text{or both})$$

$$\begin{cases}
 (2\alpha_{11} + \alpha_{21})(2\alpha_{12} + \alpha_{22}) = 0 & (i,j) = (1,2) \\
 (2\alpha_{11} + \alpha_{21})(2\alpha_{13} + \alpha_{23}) = 0 & (i,j) = (1,3) \\
 (2\alpha_{12} + \alpha_{22})(2\alpha_{13} + \alpha_{23}) = 0 & (i,j) = (2,3)
 \end{cases}$$

What are all possibilities for the above ?



4 cases (**)

- (1) $2\alpha_{11} + \alpha_{21} = 0$ $m=1$
 $2\alpha_{12} + \alpha_{22} = 0$ $n=2$
- (2) $2\alpha_{11} + \alpha_{21} = 0$ $m=1$
 $2\alpha_{13} + \alpha_{23} = 0$ $n=3$
- (3) $2\alpha_{12} + \alpha_{22} = 0$ $m=2$
 $2\alpha_{11} + \alpha_{21} = 0$ $n=1$
- (4) $2\alpha_{12} + \alpha_{22} = 0$ $m=2$
 $2\alpha_{13} + \alpha_{23} = 0$ $n=3$

$$\exists m, n \in \{1, 2, 3\}, m \neq n$$

$$\begin{aligned}
 2\alpha_{1m} + \alpha_{2m} &= 0 \\
 2\alpha_{1n} + \alpha_{2n} &= 0
 \end{aligned}$$

$$(***) \begin{cases} (2\alpha_{31} + \alpha_{21})(2\alpha_{32} + \alpha_{22}) = 0 & (i,j) = (1,2) \\ (2\alpha_{31} + \alpha_{21})(2\alpha_{33} + \alpha_{23}) = 0 & (i,j) = (1,3) \\ (2\alpha_{32} + \alpha_{22})(2\alpha_{33} + \alpha_{23}) = 0 & (i,j) = (2,3) \end{cases}$$

similarly:

4 cases

(***)

$$\Rightarrow \exists m, s \in \{1, 2, 3\} \quad m \neq s$$

$$2\alpha_{3m} + \alpha_{2m} = 0$$

$$2\alpha_{3s} + \alpha_{2s} = 0$$

$$\text{Thus } \exists m, n, s \in \{1, 2, 3\} \quad m \neq n, m \neq s$$

$$2\alpha_{1m} + \alpha_{2m} = 0$$

$$2\alpha_{1n} + \alpha_{2n} = 0$$

$$2\alpha_{3m} + \alpha_{2m} = 0$$

$$2\alpha_{3s} + \alpha_{2s} = 0$$

\Rightarrow

$$\alpha_{1m} = \alpha_{3m}$$

$$\alpha_{2m} = -2\alpha_{1m}$$

$$\alpha_{2n} = -2\alpha_{1n}$$

$$\alpha_{2s} = -2\alpha_{3s}$$

Two cases $n \neq s$ or $n = s$

Define vectors $v_m = \alpha_{1m} e_1 - 2\alpha_{1m} e_2 + \alpha_{3m} e_3$

$v_n = \alpha_{1n} e_1 - 2\alpha_{1n} e_2 + \alpha_{3n} e_3$

$v_s = \alpha_{1s} e_1 - 2\alpha_{3s} e_2 + \alpha_{3s} e_3$

so v_m, v_n, v_s are linearly independent if $n \neq s$

CASE I $n \neq s$

$l = m, j = s \quad m (*) :$

$$\alpha_{1n} \alpha_{2s} + 2\alpha_{1n} \alpha_{3s} + \alpha_{2n} \alpha_{1s} + \alpha_{2n} \alpha_{2s} + \alpha_{2n} \alpha_{3s} + 2\alpha_{3n} \alpha_{1s} + \alpha_{3n} \alpha_{2s} = 0$$

$$\cancel{\alpha_{1n} (-2\alpha_{3s})} + \cancel{2\alpha_{1n} \alpha_{3s}} - 2\alpha_{1n} \alpha_{1s} - 2\alpha_{1n} (-2\alpha_{3s}) - 2\alpha_{1n} \alpha_{3s} + 2\alpha_{3n} \alpha_{1s} + \alpha_{3n} (-2\alpha_{3s}) = 0$$

$$-2\alpha_{1n} \alpha_{1s} + 2\alpha_{1n} \alpha_{3s} + 2\alpha_{3n} \alpha_{1s} - 2\alpha_{3n} \alpha_{3s} = 0$$

$$(\alpha_{1n} - \alpha_{3n})(\alpha_{3s} - \alpha_{1s}) = \alpha_{1n} \alpha_{3s} - \alpha_{3n} \alpha_{3s} - \alpha_{1n} \alpha_{1s} + \alpha_{3n} \alpha_{1s} = 0$$

So either $\alpha_{1n} = \alpha_{3n}$ OR $\alpha_{3s} = \alpha_{1s}$



$$v_m = \alpha_{1m} (e_1 - 2e_2 + e_3)$$

$$v_m = \alpha_{1m} (e_1 - 2e_2 + e_3)$$

$$v_n = \alpha_{1n} (e_1 - 2e_2 + e_3)$$

$$v_n = \alpha_{1n} e_1 - 2\alpha_{1n} e_2 + \alpha_{3n} e_3$$

$$v_s = \alpha_{1s} - 2\alpha_{3s} e_2 + \alpha_{3s} e_3$$

$$v_s = \alpha_{1s} (e_1 - 2e_2 + e_3)$$

Contradiction, not linearly independent

(v_m, v_n) not LI

(v_m, v_s) not LI



Case 2 $n = s$

Then

$$\alpha_{1m} = \alpha_{3m}$$

$$\alpha_{1n} = \alpha_{3n}$$

$$\alpha_{2m} = -2\alpha_{1m}$$

$$\alpha_{2n} = -2\alpha_{1n}$$

So

$$v_m = \alpha_{1m} e_1 - 2\alpha_{1m} e_2 + \alpha_{3m} e_3 = \alpha_{3m} (e_1 - 2e_2 + e_3)$$

$$v_s = v_n = \alpha_{1n} e_1 - 2\alpha_{1n} e_2 + \alpha_{3n} e_3 = \alpha_{3n} (e_1 - 2e_2 + e_3)$$

contradiction, v_m, v_n not linearly independent.

