

7/18/11

①

Problem 1

Exercises for part I
Feb 8, 2011

Prop 2

$$\delta_a x = ax - xa$$

$$\delta_a(xy) = \overset{(1)}{a(xy)} - \overset{(2)}{(xy)a}$$

(1)

||?

it works ✓

$$x \delta_a(y) = x(ay - ya) = \overset{(2)}{x(ay)} - \overset{(2)}{x(ya)}$$

(2)

(3)

Prob 1, 2, 3 short
Prob 4 long

$$\delta_a(x)y = (ax - xa)y = \overset{(1)}{(ax)y} - \overset{(3)}{(xa)y}$$

Problem 2

Prop 3

$$\delta_a x = ax - xa$$

$$\delta_a[x, y] = \overset{(1)}{a(\overset{(2)}{xy} - yx)} - \overset{(3)}{(\overset{(4)}{xy} - yx)a}$$

(1)

it works ✓

$$[\delta_a x, y] = [ax - xa, y] = \overset{(1)}{(ax - xa)y} - \overset{(2)}{y(ax - xa)}$$

(2)

+

(3)

$$[x, \delta_a y] = [x, ay - ya] = \overset{(1)}{x(ay - ya)} - \overset{(2)}{(ay - ya)x}$$

(2)

Problem 3

Prop 4 $\delta_a x = ax - xa$

$$\delta_a(xoy) = a(xoy) - (xoy)a$$

$$= a(\cancel{x}y + \cancel{y}x) - (\cancel{x}y + \cancel{y}x)a$$

(1)
" ?

$$(\delta_a x)oy = (ax - xa)oy = \cancel{ax}y + y\cancel{xa} \quad (2)$$

$$x(\delta_a y) = x(ay - ya) = \cancel{xa}y + y\cancel{ya} \quad (3)$$

it works ✓

Problem 4

$$\delta_{ab} x = a_0(b_0 x) - b_0(a_0 x)$$

$$\delta_{a,b}(xoy) = a_0(b_0(xoy)) - b_0(a_0(xoy)) \quad (1)$$

$$= a(b(\cancel{x}y + \cancel{y}x) + (\cancel{x}y + \cancel{y}x)b)$$

$$+ (b(\cancel{x}y + \cancel{y}x) + (\cancel{x}y + \cancel{y}x)b)a$$

$$- b(a(\cancel{x}y + \cancel{y}x) + (\cancel{x}y + \cancel{y}x)a)$$

$$- (a(\cancel{x}y + \cancel{y}x) + (\cancel{x}y + \cancel{y}x)a)b$$

(3)

$$(\delta_{a,b}x) \circ y = (a \circ (b \circ x) - b \circ (a \circ x)) \circ y \quad (2)$$

$$= \left(a \left(\frac{b}{x} + \frac{x}{b} \right) + (b/x + x/b)a \right) y + y \left(a(b/x + x/b) + (b/x + x/b)a \right) - \left(b \left(\frac{a}{x} + \frac{x}{a} \right) + (a/x + x/a)b \right) y - y \left(b(a/x + x/a) + (a/x + x/a)b \right)$$

$$\otimes \quad x \circ (\delta_{a,b}y) = x \circ (a \circ (b \circ y)) - x \circ (b \circ (a \circ y)) \quad (3)$$

$$= x \left(a \left(\frac{b}{y} + \frac{y}{b} \right) + (b/y + y/b)a \right) + \left(a \left(\frac{b}{y} + \frac{y}{b} \right) + (b/y + y/b)a \right) x - x \left(b \left(\frac{a}{y} + \frac{y}{a} \right) + (a/y + y/a)b \right) - \left(b \left(\frac{a}{y} + \frac{y}{a} \right) + (a/y + y/a)b \right) x$$

IT WORKS

Problem 5

$$ab = [a, b]$$

Need to prove $a^2 = 0$ $(ab)c + (bc)a + (ca)b = 0$ ✓

$$a^2 = [a, a] = \cancel{a}a - a\cancel{a} = 0 \quad \text{check}$$

$$\begin{aligned} (ab - ba)c - c(ab - ba) + (bc - cb)a - a(bc - cb) \\ + (ca - ac)b - b(ca - ac) = 0 \quad \text{check} \end{aligned}$$

Prob 6 $ab = a \circ b$ NTo mwr $ab = ba$ ~~check~~

$$a(a^2b) = a^2(ab)$$

$a \circ b = ab + ba = (ba)$ check

$$a \circ ((a \circ a) \circ b) = \boxed{\begin{array}{l} \textcircled{1} \textcircled{2} \textcircled{3} \textcircled{4} \\ a(a^2b + b/a^2) + (a^2b + b/a^2)a \\ \textcircled{1'} \textcircled{3'} \textcircled{2'} \textcircled{4'} \\ a^2(a^2b + ba) + (a^2b + ba)a^2 \end{array}}$$

check

Problem 7

$$u(u^2v) = u^2(uv)$$

$$(u+w)((u+w)^2v) = (u+w)^2((u+w)v)$$

$$(u+w)((u^2+2uw+w^2)v) = (u^2+2uw+w^2)(uv+uv)$$

degree 0 in w $u(u^2v) = u^2(uv)$

degree 1 in w $w(u^2v) + u(2uwv) = u^2(wv) + (2uw)(uv) \quad (1)$

degree 2 in w $u(w^2v) + w(2uwv) = w^2(uv) + (2uw)(wv)$

degree 3 in w $w(w^2v) = w^2(wv)$

~~corrected~~ ~~input~~ ~~print~~ ~~slides~~ ~~there~~

(5)

$$(1) \quad w(u^2 v) + u(2uw)v = u^2(wv) + 2(uw)(uv) \\ v(u^2 w) + u(2uv)w = u^2(wv) + 2(uv)(uw)$$

Subtract

$$\delta_{vw}(u^2) - 2u \delta_{vw}(u) = 0$$

replace u by $x+y$

$$\delta_{vw}(x^2 + 2xy + y^2) - 2(x+y) \delta_{vw}(x+y) = 0$$

$$\delta_{vw}(x^2) + 2\delta_{vw}(xy) + \delta_{vw}(y^2) = 2x\delta_{vw}(x) + 2y\delta_{vw}(x) \\ + 2x\delta_{vw}(y) + 2y\delta_{vw}(y)$$

QED it works

Prob 1-7 are OK.