

MR0143793 (26 #1345) 17.30 22.80

Jacobson, Nathan

★**Lie algebras.**

Interscience Tracts in Pure and Applied Mathematics, No. 10

Interscience Publishers (a division of John Wiley & Sons), New York-London 1962
 ix+331 pp.

This book contains a very clear presentation of the theory of Lie algebras, from basic concepts to the deepest part of the theory. It is gratifying to see that the theory on the classification of irreducible modules is now available in such a simplified and elementary form. The author's work on the classification of simple Lie algebras over an arbitrary field is also presented in a unified manner. The book consists of 10 chapters. We shall review each chapter separately.

Chapter I: Basic concepts. Chapter II: Solvable and nilpotent Lie algebras. The theorems of Engel and Lie on nilpotent and solvable Lie algebras of linear transformations and the decomposition into weight spaces of a module are given by considering the enveloping associative algebra of the given linear Lie algebra. A characterization is given of linear Lie algebras with semi-simple enveloping associative algebras. Chapter III: Cartan's criterion and its consequences. Cartan's criteria on solvability and semi-simplicity in terms of the Killing form are given. The complete reducibility of modules over a semi-simple Lie algebra is proved by using a lemma of Whitehead. The "radical splitting" theorem of Levi and its improvement by Malcev and Harish-Chandra are given. This chapter also contains a representation theory of the split 3-dimensional simple Lie algebra, which is basic in the whole theory of Lie algebras, and the Jacobson-Morozov theorem on completely reducible linear Lie algebras.

Chapter IV: Split semi-simple Lie algebras. These algebras are completely classified. Since the algebras are assumed to be split, the algebraic closedness of the ground field is not needed. Chapter V: Universal enveloping algebra. The existence and basic properties of the universal enveloping algebra, including the Poincaré-Birkhoff-Witt theorem, are proved. Also, free Lie algebra, the Campbell-Hausdorff formula, cohomology theory, restricted Lie algebras of characteristic p , and restricted universal enveloping algebras are discussed. Chapter VI: The theorem of Ado and Iwasawa. The chapter contains a simplification of Harish-Chandra's proof of Ado's theorem, using universal enveloping algebras, and the author's simple proof of Iwasawa's theorem, which yields additional information on the complete reducibility of representations. Chapter VII: Classification of irreducible modules. A surprisingly simple proof is given for Cartan's theorem which establishes a one-to-one correspondence between irreducible modules of a semi-simple Lie algebra and dominant integral forms on a Cartan subalgebra, and similarly for the existence of semi-simple Lie algebra. Basic irreducible modules are exhibited for some simple Lie algebras.

Chapter VIII: Characters of irreducible modules. After deriving Freudenthal's recursion formula for the multiplicities, Weyl's character formula is derived purely algebraically. From this, formulas of Kostant and Steinberg are derived quickly. Chapter IX: Automorphisms. Basic facts about the automorphisms of semi-simple Lie algebras are given, and for some simple Lie algebras the groups of automorphisms are explicitly determined. {The reviewer feels that, in Theorem 4, p. 281, the group G/G_0 should be described explicitly.} Chapter X: Simple Lie algebras over an arbitrary field. After some

ground work, the determination of simple Lie algebras of the types A-D (except for D_4) over a field K is reduced to that of simple associative algebras over K . The case where K is the real field is considered in detail. {In spite of this, the reviewer feels that the book would be more useful if it contained a general theory of real semi-simple Lie algebras.}

At the end of each chapter a dozen or two well-selected exercise problems are given.

Rimhak Ree

© Copyright American Mathematical Society 2017