$\boxed{\text{P. 276}}$

§4 abelian extensions of Leibniz algebras & representations

Definition an abelian extension of Leibniz algebras is

a $\boxed{\text{short exact}}$ sequence of Leibniz algebras

$0 = \ker\alpha$ α is 1:1
$\mathrm{im}\,\beta = \mathfrak{g}$ β is onto
$\mathrm{im}\,\alpha = \ker\beta$

$$0 \longrightarrow M \xrightarrow[\mathrm{im}=\ker]{\alpha} \mathfrak{h} \xrightarrow[\mathrm{im}=\ker]{\beta} \mathfrak{g} \longrightarrow 0$$

with M an abelian Leibniz algebra (which is to say, an abelian Lie algebra, $[x,y]=0\ \forall x,y$)

$$\boxed{\mathfrak{g} \simeq \mathfrak{h}/\ker\beta = \mathfrak{h}/\mathrm{im}\,\alpha \simeq \mathfrak{h}/M}$$

This sequence allows the definition of 2 actions of $\mathfrak{g}$ on ~~$\mathfrak{g}$~~ $\mathfrak{h}$

#1 $\quad \mathfrak{g} \times M \longrightarrow M \qquad [g,m] := [\tilde{g},m]$

($\tilde{g}$ is ~~~~ a lifting of $g \in \mathfrak{g}$ to $\mathfrak{h}$ i.e. $\mathfrak{g}$ ~~~~ $\simeq$ $\mathfrak{h}/M$

M an ideal in $\mathfrak{h}$, and ~~~~ $g = \tilde{g} + M$, ~~~~ so $\tilde{g} \in \mathfrak{h}$, $m \in M \subset \mathfrak{h}$

and $[\tilde{g},m] \in M$ is the bracket in $\mathfrak{h}$)

~~#2~~

#2 $\quad M \times \mathfrak{g} \longrightarrow M \qquad [m,g] = [m,\tilde{g}]$

The Leibniz identity (L) $\quad [x,[y,z]] = [[x,y],z] - [[x,z],y]$

in the Leibniz algebra $\mathfrak{h}$ implies the relations

(MLL) $\quad [m,[x,y]] = [[m,x],y] - [[m,y],x]$

(LML) $\quad [x,[m,y]] = [[x,m],y] - [[x,y],m]$

(LLM) $\quad [x,[y,m]] = [[x,y],m] - [[x,m],y]$

(think left derivations with a twist)