<u>Definition 4.1</u> A <u>representation</u> of a Leibniz algebra $\mathfrak{g}$ is a k-module M (for us a vector space M) together with two bilinear maps $\mathfrak{g} \times M \to M$ & $M \times \mathfrak{g} \to M$ satisfying (MLL), (LML), (LLM).

<u>Remarks</u> • The first axiom (MLL) involves only the right action of $\mathfrak{g}$ on M.

• The second and third axioms imply

$$(ZD) \qquad [x,[y,m]] + [x,[m,y]] = 0$$

<u>Example</u> If $M = \mathfrak{g}$ you get the <u>adjoint</u> <u>representation</u> of the Leibniz algebra $\mathfrak{g}$.

Def 4.3 A Representation M of $\mathfrak{g}$ is <u>symmetric</u> if $[m,x] + [x,m] = 0 \quad \forall x \in \mathfrak{g}, m \in M$

and <u>antisymmetric</u> if $[x,m] = 0 \quad \forall x \in \mathfrak{g}, m \in M$

(<u>trivial</u>, if symmetric & anti-symmetric, i.e. $[x,m) = 0 = [m,x]$)

<u>Examples</u> • A representation of a Lie algebra $\mathfrak{g}$ is an example of a symmetric rep. of the Leibniz algera $\mathfrak{g}$

• $\mathfrak{g}^{ann}$ is a representation of the Leibniz algebra $\mathfrak{g}_{Lie}$ which is anti symmetric.

$\mathfrak{g}^{ann}$ is by definition the kernel of $\mathfrak{g} \to \mathfrak{g}_{Lie} = \mathfrak{g}/I$ i.e. $\mathfrak{g}^{ann} = I = \mathrm{span}\{[x,x] : x \in \mathfrak{g}$