Holgate, P.

Sequences of powers in genetic algebras.


The paper begins with a brief description of the theory of genetic algebras as it appears in the work of the reviewer [Proc. Roy. Soc. Edinburgh 59 (1939), 242–258; MR0000597 (1.99e); J. London Math. Soc. 15 (1940), 136–149; MR0002854 (2,121h); ibid. 20 (1945), 238; MR0016757 (8.63a); Quart. J. Math. Oxford Ser. 12 (1941), 1–8; MR0005111 (3,102f)], H. Gonshor [Proc. Edinburgh Math. Soc. (2) 12 (1960/61), 41–53; MR0124367 (23 #A1680); ibid. 14 (1964/65), 333–338; MR0194215 (33 #2428)], O. Reiersøl [Math. Scand. 10 (1962), 25–44; MR0137740 (25 #1189)] and the author [Proc. Edinburgh Math. Soc. (2) 15 (1966), 1–9; MR0201199 (34 #1083)]. All these writers paid attention to the sequence $S$ of plenary powers (that is, the powers obtained by repeated squaring) of an arbitrary element in a genetic algebra, but the algebras studied were associated with specific genetic situations or were in other ways restricted.

The author now studies $S$ in a more general setting, namely in an arbitrary commutative special train algebra containing an idempotent element. It is shown that $S$ forms a train, and that the distinct values among the plenary train roots, and upper bounds for their multiplicities, can be obtained by recursion on the dimension of the algebra. In conclusion the author compares his approach with the methods used by some mathematical biologists including J. B. S. Haldane [J. Genetics 22 (1930), 359–372], J. H. Bennett [Ann. Eugenics 18 (1954), 311–317; MR0061360 (15,813b)] and P. A. P. Moran [Statistical processes of evolutionary theory, Clarendon, Oxford, 1962].

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