In this paper the authors introduce the notion of a chain of evolution algebras. Let $E$ be a finite-dimensional algebra over a field $\mathbb{F}$ with basis $\{e_1, \ldots, e_n\}$ such that $e_i e_j = 0$ if $i \neq j$ and $e_i e_i = \sum_{k=1}^n a_{ik} e_k$. This algebra is called a finite-dimensional evolution algebra.

Consider a family $\{E[s, t] : s, t \in \mathbb{R}, 0 \leq s \leq t\}$ of $n$-dimensional evolution algebras over a field $\mathbb{F}$, with basis $\{e_1, \ldots, e_n\}$ and multiplication table $e_i e_i = M_i^{[s, t]} = \sum_{j=1}^n a_{ij}^{[s, t]} e_j$, $i = 1, \ldots, n$; $e_i e_j = 0$, $i \neq j$. Here parameters $s, t$ are considered as time. Denote by $M[s, t] = (a_{ij}^{[s, t]})_{i, j = 1, \ldots, n}$ the matrix of structural constants.

A family $\{E[s, t] : s, t \in \mathbb{R}, 0 \leq s \leq t\}$ of $n$-dimensional evolution algebras over a field $\mathbb{F}$ is called a chain of evolution algebras (CEA) if the matrix $M[s, t]$ of structural constants satisfies the Chapman-Kolmogorov equation

$$M[s, t] = M[s, \tau] M[\tau, t],$$

for any $s < \tau < t$.

Let $\{M[s, t] : 0 \leq s \leq t\}$ be a family of stochastic matrices which satisfies (1); then it defines a Markov process.

In this paper the authors give several examples (time homogeneous, time non-homogeneous, periodic, etc.) of CEAs. A continuum set of non-isomorphic evolution algebras is constructed. It is shown that the corresponding discrete time CEA is dense in the set. A criterion for an evolution algebra to be baric is obtained. Moreover, the concept of a property transition is given. The behavior of the baric property depending on the time is described for some concrete CEAs. Some examples of almost surely baric CEAs are constructed. For an example of two-dimensional CEA the full set of idempotent elements is described and it is shown that for some values of the parameters the number of idempotent elements does not depend on time, but for other values of the parameters there is a critical time $t_c$ such that the chain has only two idempotent elements if the time $t \geq t_c$ and it has four idempotent elements if the time $t < t_c$.

Henrique Guzzo, Jr.

References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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