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MR3081544 (Review) 11N05 11N35 11N36
Maynard, James (4-OX)
Bounded length intervals containing two primes and an almost-prime. (English summary)
Bull. Lond. Math. Soc. **45** (2013), *no. 4,* 753–764.

This paper's title must have seemed provocative in 2012: is the author claiming to have proved the existence of infinitely many bounded gaps between primes?

Nowadays, the reader is likely to conclude simply that he or she has found the wrong paper. In a subsequent, spectacular breakthrough ["Small gaps between primes", preprint, arXiv:1311.4600, Ann. of Math., to appear], the author proved precisely this, and indeed the much sharper result that

$$\liminf_{n\to\infty} \frac{p_{n+m}-p_n}{\log n} \ll m^3 e^{4m}.$$

The latter paper, along with Yitang Zhang's ([Ann. of Math. (2) **179** (2014), no. 3, 1121–1174; MR3171761], the first paper to obtain bounded gaps), has inspired a flurry of ongoing work, and any prospective reader of the present paper will almost certainly want to begin there instead.

The present paper proves results of a similar flavor, but conditionally on unproved hypotheses. Assuming the Elliott-Halberstam conjecture, Maynard proves the existence of infinitely many intervals of the type $[n, n+C(k,\theta)]$ containing two primes and k integers with at most r prime factors each. The constant $C(k,\theta)$ depends on the level of distribution assumed in Elliott-Halberstam; any $\theta > \frac{1}{2}$ will do.

For $k=4$ and $\theta = 0.99$, Maynard also proves the sharp quantitative result $C(k,\theta) = 90$, subject to an additional conjectural generalization of Elliott-Halberstam.

The author follows the so-called 'GPY method' introduced by D. A. Goldston, J. Pintz and C. Y. Yıldırım in [Ann. of Math. (2) **170** (2009), no. 2, 819–862; MR2552109 (2011c:11146)] and further investigated by the same authors and S. W. Graham in [Proc. Lond. Math. Soc. (3) **98** (2009), no. 3, 741–774; MR2500871 (2010a:11179)]. Building on technical results proved there and elsewhere, the present paper is short, reasonably simple, and quite explicit.

The reader is bound to notice the author's care and persistence in exploring the limitations and possibilities of the GPY method. To state the obvious, it paid off.

Frank Henry Thorne

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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