An evolution algebra is a finite-dimensional algebra $E$ with basis $\{e_1, \ldots, e_n\}$ and multiplication given by $e_i e_i = \sum_{j=1}^{n} a_{ij} e_j$ and $e_i e_j = 0$ ($i \neq j$). Such an algebra describes self-reproduction of non-Mendelian genetics. The elements $e_1, \ldots, e_n$ represent alleles and $a_{ij}$ is the probability that $e_i$ becomes $e_j$ in the next generation.

The authors obtain elementary algebraic properties of nondegenerate evolution algebras (i.e., $e_i e_i \neq 0$ for every $i = 1, \ldots, n$) over arbitrary fields. Evolution algebras are commutative (and hence flexible) but are not necessarily power-associative. Not every evolution algebra is a Banach algebra. However, if $E$ is a real evolution algebra such that $\sum_{j=1}^{n} |a_{ij}| \leq 1$ for every $i = 1, \ldots, n$, then $E$ is a Banach algebra. Evolution algebras are not closed under subalgebras. A subalgebra $F$ of $E$ is called an evolution subalgebra if $F = \{\sum_{i=1}^{n} \alpha_i e_i | \alpha_i = 0, \forall i \notin I\}$ for some $I \subset \{1, \ldots, n\}$. An evolution algebra is evolutionary simple if it has no proper evolution subalgebras. An evolution algebra is evolutionary semisimple if it is a direct sum of some of its evolutionary simple evolution subalgebras. Every evolution algebra $E$ decomposes as a vector space into $P(E) \oplus T(E)$, where $P(E)$ is an evolutionary semisimple evolution algebra and $T(E)$ is a subspace (not necessarily a subalgebra) of $E$ that can be made into an evolution algebra.

As stated by the authors, the algebraic properties are of importance from the biological point of view.

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