Derivations in genetic algebras.


There has been considerable interest recently in the automorphisms and derivations of genetic algebras. The author lists several papers. Others dealing specifically with the topic are by A. Micali, T. M. M. Campos, M. C. Costa e Silva and S. Ferreira [Linear Algebra Appl. 64 (1985), 175–181; MR0776524 (86e:17011); Algèbres génétiques, see pp. 71–76, Hermann, Paris, to appear] and the reviewer [Linear Algebra Appl. 85 (1987), 75–79; MR0869346 (88b:17044)]. Several recent papers devoted primarily to other topics in genetic algebra contain discussions of the derivation algebras of the structures concerned.

In this paper the author sets out to review, simplify and clarify the theory that has grown up mainly through the study of special cases. He begins by setting out general results useful in the study of derivations. Theorem 1 should read: Let $A$ be a train algebra with a nonzero idempotent in which all train roots except the first are distinct from 1. Then any derivation maps every element into an element of weight 0. He then studies derivations in graded algebras, showing under suitable conditions that every derivation can be decomposed relative to the eigenspaces of the multiplication by $c_0$, a nonzero element belonging to the component $C_0$ of the grading, which has been supposed one-dimensional. Theorem 3 gives relations between derivations and mappings into the set of strong nilpotents (= annihilators, absolute divisors of zero) of the algebra. The author then goes on to obtain in a simple and direct way, results of R. C. F. Costa [Bol. Soc. Brasil. Mat. 13 (1982), no. 2, 69–81; ibid. 14 (1983), no. 1, 63–80; MR 85i:17031a] and Micali et al. [Comm. Algebra 12 (1984), no. 1-2, 239–243; MR0732196 (85g:17014)]. He then demonstrates the computational power of the simplification that he has introduced by studying the algebras for polyploidy with multiple alleles and mutation; and for sex linkage where the fundamental algebra is not genetic (in what is usually called “the sense of Gonshor”) but is graded. Finally, he discusses the derivations of algebras for several loci with recombination.

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