A special train algebra is a commutative algebra for which there exists a basis $a_0, a_1, \ldots, a_n$ with a multiplication table of the following kind: $a_i a_j = \sum x_{ijk} a_k$, where

1. $x_{000} = 1$;
2. for $k < j$, $x_{0jk} = 0$;
3. for $i, j > 0$ and $k < \max(i, j)$, $x_{ijk} = 0$;
4. all powers of the ideal $(a_1, a_2, \cdots, a_n)$ are ideals.

The $x_{0jj}$ (abbreviated $\lambda_j$) are called the train roots of the algebra. They are the characteristic roots of the operator which is defined by multiplication by $a_0$. Necessarily $\lambda_0 = 1$. The author proves that every special train algebra which has no train root satisfying $2\lambda = 1$ has a unique non-zero idempotent, and applies this result to a number of special train algebras suggested by genetic situations.

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