
An \( n \)th-order Bernstein algebra is an algebra \( A \) over a field \( F \) (in this paper \( \text{ch} F = 0 \) is assumed), having a nonzero homomorphism of algebras \( \omega: A \to F \), such that \( x^{[n+2]} = (\omega(x))^{2^n} x^{[n+1]} \) for every \( x \in A \). Here \( x^{[n]} \) denotes the \( n \)th-plenary power of the element \( x \).

The existence of a nonzero idempotent element \( e \) and the associated Peirce decomposition is known for every \( n \)th-order Bernstein algebra. So \( A = Fe + U_e + Z_e \), where \( U_e = \{ u \in \text{Ker} \omega \mid eu = \frac{1}{2} u \} \) and \( Z_e = \{ z \in \text{Ker} \omega \mid L_e^2(z) = 0 \} \). The dimensions of the \( U \)- and the \( Z \)-component in a Peirce decomposition do not depend on the particular idempotent element.

A derivation \( D: A \to A \) is given by a triple \( (\tilde{u}, f, g) \), where \( \tilde{u} = D(e) \in U_e \) and \( f: U_e \to U_e \) and \( g: Z_e \to Z_e \) are linear maps that satisfy some conditions.

In this paper authors study derivations in the power-associative case and for \( n \)th-order Bernstein algebras satisfying \( (U_e \oplus Z_e)^2 \subseteq Z_e \), finding some properties and some upper bounds for the dimension of the derivation algebra. Also, inner derivations in the two mentioned cases are studied, and conditions over an element \( x \) that guarantee that the multiplication by \( x \) is a derivation of \( A \) are found. In particular, \( \text{Inn}(A) \) is explicitly given for Bernstein algebras of order \( n \) that are also Jordan, in which case \( \text{Der}(A) \) is proved to be nonzero.