Train algèbres alternatives: relation entre nilindice et degré. (French. English summary) [Alternative train algebras: relation between nilindex and degree]

In this paper train algebras are considered. These algebras appear in a biological context, and so they are baric algebras. A baric algebra $A$ over a field $K$ is a $K$-algebra with a weight homomorphism $\omega: A \to K$, that is, a nonzero algebra homomorphism. In general, algebras appearing in genetics are non-associative, but commutative.

A train algebra of degree $n+1$ is a baric algebra $(A, \omega)$ satisfying
\[
x^{n+1} = \sum_{k=1}^{n} \alpha_k \omega(x)^{n-k+1} x^k,
\]
for every $x \in A$.

Here, $x^k$ denotes the left principal power of $x$; that is, $x^{k+1} = L_x^k x$, where $L_x: A \to A$ denotes the usual left multiplication.

In a previous paper, the first author studied associative train algebras, proving that the only train identities of degree 3 are $x^3 = \omega(x)x^2$ and $x^3 = 2\omega(x)x^2 - \omega(x)^2x$ and the ones of degree 4 are $x^4 = \omega(x)x^3$, $x^4 = 2\omega(x)x^3 - \omega(x)^2x^2$, $x^4 = 3\omega(x)x^3 - 3\omega(x)^2x^2 + \omega(x)^3x$.

In the present paper the associativity assumption is removed. Instead, the authors consider power-associativity (an algebra is power associative if the subalgebra generated by one element is associative). So the main result of the paper proves that, assuming $\text{char} K \neq 0$, the only polynomials associated to power-associative train algebras are of the type $x^p(x-1)^{n-p}$, with $0 \leq p \leq n-1$. The existence of a nonzero idempotent element is also proved.

If the algebra considered is alternative (recall that an alternative algebra can be characterized as an algebra in which every two elements generate an associative algebra), then using the previously proved existence of a nonzero idempotent and the associated Peirce decomposition, $A = A_{00} + A_{01} + A_{10} + A_{11}$, the following result can be proved:

Theorem. If $A$ is a train algebra of polynomial $x^p(x-1)^{n-p}$, then $U_{00}$ is nil of index $\leq p+1$ and $U_{11}$ is nil of nilindex $\leq n-p$. And, conversely, if at least one among the subspaces $U_{ij}$ is zero, $U_{00}$ and $U_{11}$ are nil of nilindex $r, s$ respectively, then $A$ is a train algebra of degree $r + s$.

Consuelo Martínez

References


*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*

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