Bernstein algebras whose lattice of ideals is distributive.


In this paper the authors characterize Bernstein algebras whose lattice of ideals is distributive. Firstly, cases of normal, exclusive and Jordan-Bernstein algebras are considered.

If $B = K e + U e + Z e$ denotes a Bernstein algebra, the results obtained in the three mentioned cases are: (1) If $B$ is normal (that is, $U e Z e = Z e^2 = 0$) and its lattice of ideals is distributive, then either $B$ is trivial or $B = K e + K u + K z$ with $u^2 = z$. (2) If $B$ is exclusive (that is, $U e^2 = 0$), then: (i) If $B$ is a nontrivial algebra and $U e Z e = 0$, then the lattice of ideals is distributive if and only if $B = K e + K u + K z$ with $z^2 = u$. (ii) If $U e Z e \neq 0$, then the lattice of ideals of $B$ is distributive if and only if $U e$ is a cyclic $\varphi$-module, where $\varphi = R e | U e$. (3) If $B$ is Jordan-Bernstein and has a distributive lattice of ideals, then $B$ is exclusive or normal.

Results in the Jordan-Bernstein case inspire and are used for the general case, where two possibilities appear for a Bernstein algebra with a distributive lattice of ideals: (a) $\dim B = \dim B^2$, that is, $B$ is nuclear, (b) $\dim B = \dim B^2 + 1$.

It is proved that in both cases $B$ is either exclusive or normal, so its structure is known from a previous study.

{For the entire collection see MR1751123 (2001a:17002)}

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