The commutative [resp. noncommutative] duplicate $D(A)$ [resp. $D^*(A)$] was defined by I. M. H. Etherington [Proc. Edinburgh Math. Soc. (2) 6 (1941), 222–230; MR0005113 (3,103b)]. His definition was relative to a basis $a_0, \cdots, a_n$ of $A$, and, e.g., $D(A)$ comprised the linear combinations over the field of $A$ of the unordered pairs $(a_i, a_j)$, with multiplication rule $(a_i, a_j)(a_k, a_l) = (a_ia_ja_k, a_ia_ja_l)$. H. Gonshor [ibid. (2) 17 (1970/71), 289–298; MR0302218 (46 #1371)] gave a basis-free definition, emphasising that the underlying space of $D(A)$ is $A \otimes A/I$, where $I$ is the ideal generated by elements $\{x \otimes y - y \otimes x\}$. The present authors provide a rigorous definition of $D(A)$, exhibiting it as a semidirect product of $A^2$ by $N(A)$, the kernel of the canonical mapping $M: D(A) \rightarrow A^2$ given by $x \cdot y \rightarrow xy$, where $x \cdot y$ is the symmetric product. The arbitrariness of the basis definition is thus displaced onto the choice of a factor set in the extension from $A^2$ to $D(A)$ by $N(A)$. Since $N(A)$ is a trivial algebra, this makes it clear that the important properties of $D(A)$ are determined by $A^2$, not by $A$. The authors use this fact to sharpen some well-known theorems. For example, if $A^2$ is a basic algebra, or a Schafer generic algebra, then so is $D(A)$. In characteristic $\neq 2$, if $A$ is a Bernstein algebra, $D(A)$ is a special train algebra with a specified train equation. The authors then ask under what conditions, if $A$ belongs to some class $C$ of algebras, does $D(A)$ also belong to $C$? They show that it does if $A^2$ belongs to $C$, and the function $Q$ giving the factor set of the extension is a 2-cocycle of $A^2$ with coefficients in $N(A)$. This is applied to associative and Jordan algebras. Next, the relation between the derivation algebras of $A^2$ and those of $D(A)$ is studied. If $A = A^2$, they are isomorphic under wide conditions. The Bernstein case is studied in detail. Finally, there are sections on automorphisms, duplication as a functor, and on results that hold for noncommutative, but not for commutative duplicates.

P. Holgate

© Copyright American Mathematical Society 1992, 2015