A Bernstein algebra is an algebra that is not necessarily associative, which admits a one-dimensional representation \( x \rightarrow \omega(x) \), and which satisfies the identity \( (x^2)^2 = \omega^2(x)x^2 \). Such algebras arise in genetics when the hereditary mechanism is such that, as in the Hardy-Weinberg law, equilibrium is reached after a single generation of breeding by random mating.

Genetic algebras tend to lie outside the mainstream of nonassociative algebras, which is characterised by “nearly associative” substitutes for the associative law. There is therefore some interest in exploring those algebras that as well as being genetic, of a particular type, belong to one or other of the more commonly studied classes of nonassociative algebras. The main theorem of this paper is that the following three statements are equivalent: (a) \( A \) is a Jordan Bernstein algebra, (b) \( A \) is a power-associative Bernstein algebra, (c) the identity \( x^3 = \omega(x)x^2 \) holds in \( A \). The author then examines the properties of Jordan Bernstein algebras. He shows that there is always a basis consisting of eigenvectors of the multiplication by a specified element of weight 1. He obtains the result that if in a Bernstein algebra the kernel of \( \omega \) is nilpotent, then it is a train algebra.

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