8
GORDONIAN LOGIC

If you ask a casual sudoku solver what the rules to the puzzle are, chances are that the answer you get will explain that every row, column, and 3×3 box must contain the numbers 1 to 9, and that will be it. As noted earlier, there are a few other rules. One states that you can have no more than 30 given numbers at the start. Another says that the cells that are occupied with numbers at the start need to be symmetric (so if the upper left cell has a number in it, the lower right cell must have one, too). Another rule (not followed by everyone) is that you should be able to reach the solution through logical means. And the final rule is that the puzzle can have only one possible answer. That rule is the key to Gordonian Logic.

GORDONIAN RECTANGLES

Early on in the sudoku craze, Frank Longo, who wrote all the examples and puzzles in this book, sent me a puzzle that he claimed was not solvable without guessing. I love a challenge, so I gave it a shot. The starting grid is shown, labeled Example 17. I was breezing through the puzzle, thinking to myself that the puzzle was for beginners, until I got to the point shown in Example 17-1.

Example 17

		8	4	1	3	5		
	2	5						3
			3	7			2	9
8	6			2	1			
5						1	6	
		9	6	5	8	7		

Example 17-1

6	9	8	4	1	3	5	7	2
37	34	47	257	69	2579	69	8	1
1	2	5	78	689	79	69	4	3
4	5	1	3	7	6	8	2	9
9	7	2	58	48	45	3	1	6
8	6	3	9	2	1	4	5	7
5	34	47	27	349	2479	1	6	8
37	8	6	1	34	47	2	9	5
2	1	9	6	5	8	7	3	4

I wasn't stumped yet, though. The cells 24 and 26 are the only ones in row 2 that have 2's and 5's as candidates, so they form a hidden pair. The other possibilities can be removed from those cells. And in row 8, the 4 has to be in either cell 85 or 86, so the 4's can be removed from the candidate lists of cells 75 and 76. That leaves this grid:

Example 17-2

6	9	8	4	1	3	5	7	2
37	34	47	25	69	25	69	8	1
1	2	5	78	689	79	69	4	3
4	5	1	3	7	6	8	2	9
9	7	2	58	48	45	3	1	6
8	6	3	9	2	1	4	5	7
5	34	47	27	39	279	1	6	8
37	8	6	1	34	47	2	9	5
2	1	9	6	5	8	7	3	4

I then stared at this grid for a long, long time. Finally I had a breakthrough that revolutionized my sudoku solving. It was a method that turned hard-to-solve puzzles into mere pushovers. I dubbed it Gordonian Rectangles. Here's how it works. Suppose that cell 35 isn't an 8. You'd then have this as part of the grid:

Example 17 With No 8 at Cell 35

				69		69		
				69		69		

With no 8 at cell 35, how would you finish this puzzle? No matter what other numbers you put in, you'd never have anything to tell you where the 6's and 9's went in cells 25, 27, 35, and 37. Rows 2 and 3, columns 5 and 7, and boxes 2 and 3 each have pairs with the numbers 6 and 9 in those four cells, so nothing will determine if the 6's go in 27 and 35 and the 9's in 25 and 37, or vice versa. In other words, without the 8 in cell 35, there would be two valid answers. But since the puzzle was tested by Frank's computer, I knew that it had to have just one valid solution, so I reasoned that if the 8 weren't in cell 35, the rest of the numbers would necessarily form an impossibility. They had to, or else the two possible ways to place the 6's and 9's shown below and on the next page would create two valid answers. So, if not having an 8 in cell 35 meant there would be an impossible situation somewhere in the puzzle eventually, then an 8 necessarily had to go in cell 35. I confidently filled in the 8. The diagram is shown in Example 17-3, with the 8's removed from cell 35's buddies.

Example 17 With No 8 at Cell 35 Answer 1

				6		9		
				9		6		

Example 17 With No 8 at Cell 35 Answer 2

				9		6		
				6		9		

Example 17-3

6	9	8	4	1	3	5	7	2
37	34	47	25	69	25	69	8	1
1	2	5	7	8	79	69	4	3
4	5	1	3	7	6	8	2	9
9	7	2	58	4	45	3	1	6
8	6	3	9	2	1	4	5	7
5	34	47	27	39	279	1	6	8
37	8	6	1	34	47	2	9	5
2	1	9	6	5	8	7	3	4

Once that 8 is placed, the puzzle can be solved on autopilot.

Example 17 Answer

6	9	8	4	1	3	5	7	2
3	4	7	5	6	2	9	8	1
1	2	5	7	8	9	6	4	3
4	5	1	3	7	6	8	2	9
9	7	2	8	4	5	3	1	6
8	6	3	9	2	1	4	5	7
5	3	4	2	9	7	1	6	8
7	8	6	1	3	4	2	9	5
2	1	9	6	5	8	7	3	4

Let's look at another:

Example 18

	3	4			5			8
						4	9	1
				2			6	
	1		5					7
		2	1	7	4	6		
5					3		1	
	6			4				
3	8	1						
4			7			5	3	

After using the methods we know, this is where we get stuck:

Example 18-1

19	3	4	69	169	5	27	27	8
2	5	6	8	3	7	4	9	1
19	7	8	4	2	19	3	6	5
6	1	3	5	89	289	289	4	7
8	9	2	1	7	4	6	5	3
5	4	7	269	689	3	289	1	29
7	6	5	3	4	29	1	8	29
3	8	1	29	5	6	279	27	4
4	2	9	7	18	18	5	3	6

Can you spot where the Gordonian Rectangle is? Unlike X-wing and its cousins, Gordonian Rectangles are simple to see. Within one box, you need to have two cells that have the same pair of numbers in either the same row or the same column. Then you need to look to see if you can form a rectangle with one corner having the same pair, and another with that pair plus a third number. In this diagram, cells 11 and 31 form the start of a Gordonian Rectangle, and cell 36 has the same pair, but cell 16 would have to have three candidates including 1 and 9 for this to qualify, but instead it's already known to be a 5. Cells 95 and 96 start off a potential Gordonian Rectangle, but there is no other cell with the pair of 1 and 8 as possibilities in column 5 or 6. The only potential one left is with the cells at 17 and 18, and sure enough, cell 88 has the same two candidates, while cell 87 has those two plus one more. If cell 87 were not a 9, we'd have two solutions if the rest of the puzzle worked, and we know that can't be, so we know that cell 87 must be a 9. Once that 9 is placed, the rest of the numbers fall into place.

Example 18 Answer

1	3	4	9	6	5	7	2	8
2	5	6	8	3	7	4	9	1
9	7	8	4	2	1	3	6	5
6	1	3	5	9	2	8	4	7
8	9	2	1	7	4	6	5	3
5	4	7	6	8	3	2	1	9
7	6	5	3	4	9	1	8	2
3	8	1	2	5	6	9	7	4
4	2	9	7	1	8	5	3	6

GORDONIAN RECTANGLES PLUS

It didn't take long to realize that if the corner of the rectangle with the third candidate also had additional candidates, you could still get somewhere using Gordonian Rectangles. My sudoku buddy and coauthor Frank Longo dubbed this Gordonian Rectangles Plus. Here's an example:

Example 19

			3		4	1		
3			2	9				6
7						3		
	6				3	2		
			7		5			
		7	1				4	
		2						5
6				1	9			8
		5	6		7			

After doing all the stuff we've already learned, this is what you have:

Example 19-1

89	59	6	3	7	4	1	58	2
3	45	48	2	9	1	578	578	6
7	2	1	8	5	6	3	9	4
15	6	48	9	48	3	2	15	7
2	14	9	7	468	5	68	1368	13
58	3	7	1	68	2	568	4	9
19	19	2	4	3	8	67	67	5
6	7	3	5	1	9	4	2	8
4	8	5	6	2	7	9	13	13

The Gordonian Rectangle Plus is in cells 58, 59, 98, and 99. We can't tell if cell 58 is a 6 or an 8, but we know for certain that it can't be 1 or 3, since that would mean two valid answers. Once we eliminate 1 and 3 from cell 58's candidates, we have a pair. Both cell 57 and 58 have 6 and 8 as their only candidates, so one must be 6 and the other must be 8. That means that cell 55 can't have a 6 or 8 in it, so it must be a 4. Putting that 4 in reduces the puzzle to a gimme.

Example 19 Answer

9	5	6	3	7	4	1	8	2
3	4	8	2	9	1	7	5	6
7	2	1	8	5	6	3	9	4
5	6	4	9	8	3	2	1	7
2	1	9	7	4	5	8	6	3
8	3	7	1	6	2	5	4	9
1	9	2	4	3	8	6	7	5
6	7	3	5	1	9	4	2	8
4	8	5	6	2	7	9	3	1

SOME WARNINGS

You need to be careful when you're looking for Gordonian Rectangles to remember that the four corners must fall in only two boxes. Here is a setup that is *not* a Gordonian Rectangle:

Not a Gordonian Rectangle

		26	26					
		26	267					

This is not a Gordonian Rectangle because the four corners of the rectangle are in four different boxes. Cell 54 doesn't necessarily need to be a 7. It *could* be a 7, but it could just as easily be a 2 or 6. When the rest of the cells are filled in, it's not possible to swap the contents of the rectangle's corners and still keep the solution valid.

Another concern is that the solution may not be unique. How do you know when sitting down with a puzzle that there is only one answer? Well, technically, you don't. But you also don't know that the puzzle will work at all. The fact that it's in a newspaper and that only one answer will appear the next day, or it's in a magazine or book with a single answer in the answer section tells you that it's presumably been checked to make sure there is just one possible answer. What's interesting is that the programmers can't use Gordonian Rectangles when testing their puzzle unless they have first checked some other way to see if the puzzle has a unique solution. Online sites that automatically solve puzzles, like sudokusolver.co.uk, never *assume* that a puzzle has a unique solution, since they are used by sudoku writers to make sure their hand-generated puzzles are unique, so they often can't solve puzzles involving Gordonian Rectangles (the three puzzles above couldn't be solved by the online solver) unless you allow it to guess and then check the solution.