

3/14/18 II (1)

Prop 2.1 (c) we know that $Tu = fu$
for all minimal partial isometries in C

Each $x \in S(C)$ can be written $x = \sum_{i=1}^m \lambda_i e_i$

$e_1, \dots, e_m$ orthogonal minimal partial isometries

$$0 < \lambda_j \leq \|x\| = 1 \quad (\lambda_j = 1 \text{ for some } j)$$

$$\text{Write } x = u_1 + \dots + u_k + \sum_{j=k+1}^m \lambda_j u_j = u + \sum_{j=k+1}^m \lambda_j u_j$$

$$0 < \lambda_j < 1$$

Consider $\{u_{k+1}, \dots, u_m\}$ and let E be the triple system in C
generated by $\{u_{k+1}, \dots, u_m\}$. E is triple isomorphic to

a finite dim'l unital C^* -algebra of dimension $m-k$

and $\sum_{j=k+1}^m \lambda_j u_j \in E$ with $\|\sum_{j=k+1}^m \lambda_j u_j\| < 1$.

By the Russo-Dye theorem $\sum_{j=k+1}^m \lambda_j u_j = \frac{1}{m_1} (w_1 + \dots + w_{m_1})$

$w_1, \dots, w_{m_1}$ are unitary operators in the C^* -algebra E

let $\tilde{w}_j = u + w_j$ $1 \leq j \leq m_1$ so that

$\tilde{w}_1, \dots, \tilde{w}_{m_1}$ are partial isometries in C

$$x = \frac{1}{m_1} (\tilde{w}_1 + \dots + \tilde{w}_{m_1})$$

Thus $\tilde{w}_1, \dots, \tilde{w}_{m_1}, x$ lie in a convex subset of $B(C)$

by Lemma 3 $f(x) = \frac{1}{m_1} (f(\tilde{w}_1) + \dots + f(\tilde{w}_{m_1})) = \frac{1}{m_1} (T(\tilde{w}_1) + \dots + T(\tilde{w}_{m_1})) =$

$$\begin{aligned} & T\left(\frac{1}{m_1} \sum_{j=1}^{m_1} \tilde{w}_j\right) \\ &= T(x) \quad \square \end{aligned}$$