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Theorems 1-4 concern pulse processes.
Theorems 5, 7 concern evolution algebras

Theorem 1 (Roberts Theorem 3)

Autonomous pulse process $\Rightarrow$ $P(t) = P(0)A^t \quad (t \geq 0)$

A is the adjacency matrix for the digraph

Theorem 2 (Roberts Theorem 4)

If λ is eigenvalue of A and $|\lambda| > 1$ then the digraph D is pulse unstable for some simple pulse process

Theorem 3 (Roberts Theorem 5) (characterization of pulse stability)

(i) D is pulse stable under all autonomous pulse processes

(ii) . . . . simple (autonomous) " "

(iii) All eigenvalues of A satisfy $|\lambda| \leq 1$ and if an eigenvalue λ satisfies $B_j(\lambda) \neq [\lambda]$ (1×1 matrix) then $|\lambda| < 1$

Theorem 4 (Roberts Theorem 6) (characterization of value stability)

(i) D is value stable under all autonomous pulse processes

(ii) D . . . . simple (autonomous) " " "

(iii) D is pulse stable under all simple pulse processes and $\lambda = 1$ is not an eigenvalue of A.

Theorem 5 An evolution algebra has a unit if and only if it is a non-zero trivial evolution algebra (structure matrix diagonal & non-singular)

Theorem 7 (multiplicative spectrum in evolution algebra)

If A is an evolution algebra with structure matrix $[w_{ij}]$, if $a = \sum \alpha_i e_i \in A$, then for $\lambda \neq 0$

$$\lambda \in \sigma_m^A(a) \Leftrightarrow \lambda \text{ is an eigenvalue of } \begin{bmatrix} w_{11} \cdots w_{1n} \\ \\ w_{n1} \cdots w_{nn} \end{bmatrix} \begin{bmatrix} \alpha_1 & 0 & \cdots & 0 \\ 0 & \alpha_2 & \cdots & 0 \\ 0 & \cdots & & \alpha_n \end{bmatrix}$$