Chapter 8

Fuzzy Logic, Sharp Logic, Frames, and Bigger Pictures

8.1 Sharp (Aristotelian) Logic: A Standard Syllogism

Many elementary mathematics books with an audience similar to that of this book have a section on logic, the logic of Aristotle.[1] Thus a triplet of statements called a *syllogism*, soon appears, such as:

$$(Assumption)\ Hypothesis\ :\ All\ humans\ are\ mortal. \tag{8.1}$$
$$(Assumption)\ Hypothesis\ :\ All\ mathematicians\ are\ human. \tag{8.2}$$
$$Conclusion\ :\ All\ mathematicians\ are\ mortal. \tag{8.3}$$

The syllogism is then illustrated with some so-called set theoretic diagram:

FIGURE 8.1: Diagram of a Syllogism

The set (box) of mathematicians is contained in the set (box) of humans is contained in the set (box) of mortals. Thus by the "transitivity of containment," an assumption in itself, we get that the set (box) of mathematicians is contained in the set (box) of mortals, cf. Figure 8.1.

Like most mathematics applied to an actual situation, Aristotle's logic is an abstract idealization, an oversimplification of reality at best – a mistake at worst.[2] I do not propose tossing out Aristotelian/sharp logic, in fact we will use sharp logic in creating the proofs of II. Such logic will remain a useful

[1] The Greek philosopher Aristotle (384 B.C.–322 B.C.) was a student of Plato and teacher of Alexander the Great.

[2] Which came first the chicken or the egg? This question becomes tractible if you abandon the implicit oversimplification.

tool for tackling problems that can be posed, at least approximately, in a sharp way. In fact, a common tool used by scientists is to look at idealized, oversimplified models to better understand whatever they are dealing with. Complications are added as it becomes possible to understand them. In the next section I will point out with a simple example, cf., Figure 8.2, some of the difficulties with applying sharp, "yes-no," "black-white" logic in the real world, and why fuzzy logic is necessary.

Before that, however, I want to review very briefly a couple concepts closely associated with syllogisms and "if ..., then ..." arguments/proofs. The following exercise will be useful in II and III. If you have not read Section 2.1 and done Exercise 2.2, do so now.

Exercise 8.1 The Concepts of Necessary and Sufficient

(i) Discuss the concept of *necessary* as it is used in precise logical reasoning. Also discuss the concept of *sufficient* as it is used in precise logical reasoning.

(ii) How does a statement of the form "If A is true, then B is true." relate to the concepts of necessary and sufficient?

(iii) Can you think of any statements P and Q such that in order for P to be true it is necessary that Q be true?

(iv) Can you think of any statements P and Q such that in order for P to be true it is sufficient that Q be true?

(v) Suppose for a statement A to be true it is necessary and sufficient for B to be true. What might you say about statements A and B?

(vi) Can you think of an example of two statements A and B such that for A to be true it is necessary and sufficient for B to be true?

(v) Discuss the truth of the following statement: In order to take effective action it is necessary to have accurate information.

(vi) History and Public Policy Professor Alexander Keyssar says in [347]: *"The current state of American politics makes clear that universal suffrage is a necessary but not sufficient condition for a fully democratic political order."* Discuss.

Explain what he might mean.

(vii) Charles A.S. Hall et al., in [270, p. 40], says: *"In other words, the availability of free energy*[3] *is a necessary, but not a sufficient, condition for the availability of labor, capital and technology."*

8.2 Measuring Truth Values: Fuzzy/Measured Logic

Aristotelian logic can be refined and expanded to better handle some of the "fuzzy" problems that come up in fields like history, sociology, biology, medicine, economics, engineering .... This expanded logic is called fuzzy

[3]Free energy here means that the energy is available and capable of doing useful work, at least for human purposes. Heat energy from a forest fire is, for example, not usually available to do useful work for humans. Fossil fuels, such as gasoline in the tank of a car, have some free energy.

logic, a name which is in some ways misleading. Mathematicians are not traditionally focused on public relations or marketing; the "fuzzy" in fuzzy logic does not mean muddleheaded. Fuzzy logic is more precisely called *measured logic*. What is being measured is the truth value of statements, and the measure or truth value of a statement can be any number between 0 for not true, i.e., (completely) false, and 1 for (completely) true. (Review Section 1.4, for example.) In Aristotle's logic statements are either true or false, and statements which are half-true and half-false, for example, are not allowed, even though such exist in Nature, cf., Exercise 8.2.

Why is Sharp Logic Often a Tool Too Blunt? Let's take a really close look at the following Figure 8.2.

FIGURE 8.2: The Fuzzy Boundary Between Inside and Outside

When I say close look I really mean it, use at least a magnifying glass, better yet a microscope – even an electron microscope. If you really do this the clear boundary between points inside the box and points outside the box melts away into a discontinuous bunch of different molecules making up the ink "lines" of the box. The perfect box boundary of zero thickness exists only as an abstract, idealized, sharp approximation of reality that we imagine in our minds.

So what does it mean for a point to be either inside or outside of the box? There are points clearly well inside the box, I can assign a truth value of 1 to the statement that such a point is inside. There are points that are clearly well outside the box, I can assign a truth value of 0 to the statement that such points are inside. If I were a bit smaller than the size of an atom and started walking from the inside of the box toward the outside it would be like walking through a "forest" where the "trees" were scattered molecules of ink. I could pass through the boundary without touching a single tree. I could make estimates and measurements and the truth value of "I am inside" would fall from a value of 1 to 0, attaining values strictly between 1 and 0 along the way.[4]

Are Some Definitions Fuzzy? What is a Tree? What is a Forest? A most prominent problem with Aristotle is not really his fault: What do the words in a syllogism mean? Most real life words and the concepts they stand for are fuzzy. I once was lucky enough to be sitting in the shade of a baobab tree

[4]Whether the set of values between 1 and 0 actually achieved is all of those numbers or a discrete "quantized" subset is an interesting question for quantum physicists. I will leave this for you to ponder.

in Zimbabwe looking for large mammals and reading a book on trees, [515]. I was struck by the following quote: *"The first, and perhaps the most contentious question that arises is, 'What is a tree?' This is a very difficult, indeed impossible question to answer for a 'tree' is a popular concept and not a scientific entity."*

There are plants that 100% of botanists agree are 100% tree. There are plants that 100% of botanists agree are 100% *not* tree. Then there are plants in between that some botanists consider to be trees and others do not, with percentages varying with the plant in question. I consider "tree" to be a concept which, although fuzzy, is more scientific than many other everyday notions. But perhaps you chalk up the fuzziness to the general level of imprecision in biology; surely things are more exact in, say, physics.

We Can Measure Something We Really Cannot Define. Guess what? Things are a bit fuzzy in physics as well. What is energy? It is a concept in physics which we will study in some detail later on. The physicist, Richard P. Feynman, said: *"It is important to realize that in physics today, we have no knowledge of what energy* ***is.*** *We do not have a picture that energy comes in little blobs of a definite amount."* See [187, p. 4 - 2 (second page of Chapter 4)].

Physics does tell us, however, that there are sharp mathematical formulas for the various types of energy and that energy can be accurately measured (with, of course, some fuzzy error). Abstract energy calculations can be done that are very precise, but only because energy is tantamount to being an undefined term of a mathematical system.

You might say that the inability to define what energy is is not the same as being fuzzy. You would be correct. It is a rather strange situtation where scientists can measure physically but not philosophically. A clearer example of a fuzzy definition occurs in astrophysics with the attempt to define *planet*. In 2006 the popular press reported widely on the controversy regarding the planethood of Pluto.

Mix Biology and Politics and Things Can Get Very Fuzzy. Let's get back to biology – and political science. The United States Congress was debating the reauthorization and/or gutting of the Clean Water Act during 1995, and a part of this act deals with wetlands. In May of 1995 a committee of the National Academy of Sciences, chaired by a biologist from the University of Colorado, completed a $550,000 two-year study which made recommendations on how to define *wetlands*. The time and money involved indicate that the concept of wetlands is at least as fuzzy as the concept of tree. The people in Congress then took the definition and made it fuzzier, since they took into account not only science but the desire of some people to make money from wetlands – people some of whom also make sizeable campaign contributions.[5]

[5]What percentage of wetlands in the United States of 1776 no longer exist? In 1997 the following Web site of the EPA gave data on wetlands losses in the United States from 1780 to 1980: http://www.epa.gov/indicator/wetloss.html#chart1. A small project might consist of trying to find the information that was on this Web page since it was removed. Also, *follow the money*. Research the amount of: (1) money spent preserving wetlands in

Mixing money, power, science and politics indeed complicates the definition of the single word – wetlands – beyond belief. Such is in real life more common than Aristotelian sharpness. Mental models that claim to be meaningful need to take all of these complications into account.

Essential Habitat Definitions. Of potentially great significance is a debate that has been going on among conservation biologists for some years about how we should define an ESU, an Evolutionary Significant Unit.[6] This debate has been long and intense regarding endangered salmon species, for example, in the Pacific Northwest of the United States. An ability to look in detail at the DNA in the cells of organisms provides a great deal of extra information for the debate. A species without habitat, however, is not viable; and preservation of habitat for a given species (which is also desired by humans) is the most common obstacle in getting a political consensus to save a particular form of life.

Aldo Leopold made famous the following words in his classic essay "Round River" in *A Sand County Almanac*: *"The last word in ignorance is the man who says of an animal or plant: 'What good is it?' If the land mechanism as a whole is good, then every part is good, whether we understand it or not. If the biota, in the course of aeons, has built something we like but do not understand, then who but a fool would discard seemingly useless parts? To keep every cog and wheel is the first precaution of intelligent tinkering."*

Reductionist training that leads us, including me, to look at "the parts" that make up the whole sometimes makes us less capable of seeing the whole in our mind. As we have seen (and will see again), a complex system is one for which we cannot simply say that the whole is greater than the sum of its parts: a complex system is *different* than the sum of its parts, sometimes remarkably so. We may miss the boat entirely by not saving the whole of complex ecosystems. And seriously asking "What is it (a form of life) worth? is an indication of human-centered hubris and ignorance.

By the way, if you want a real challenge, try to define *life*. Define what it is, not just what it does, although the latter is sufficiently challenging. The language with which we do most of our logical reasoning is fuzzy at best and at times nearly meaningless when compared to the important fundamental of staying alive.

For further reading of a popular nature on fuzzy logic, cf., [365]. A serious mathematics book which clearly shows that fuzzy logic is a marriage of logic and measure theory, cf., [358]. See also [596].

---

the U.S.A. in some time period; (2) money made by turning wetlands into something else during the same time period; (3) money in the form political campaign contributions related to wetlands during the same time period. Wetland habitat is critcal for what species? Are any threatened, endangered, extinct?

[6]Susan Milius, "What's Worth Saving? A fracas over a biological term could have huge consequences for conservation", *Science News*, Vol. 158, October 14, 2000, pp. 250-252.

Exercise 8.2 Fuzzy Logic in Real Life

(i) Someone says, "I have only one vote, it does not count." Criticize this statement from the perspective of fuzzy logic.

(ii) Someone says, "The city council should not concern itself with foreign policy, because the city can do nothing about it." Criticize this statement from the perspective of fuzzy logic.

(iii) Look at VII and analyze the government's definition of "the poverty line" from the perspective of fuzzy logic. Could fuzzy logic have a noticeable impact on people's lives were it actually used by the government to define poverty?

(iv) Someone says, "There are two sides to that argument." Critique this statement from the perspective of fuzzy logic.

(v) Try classifying all humans into male and female, or male and not male. Is there any fuzziness in this situation?

(vi) Try classifying all objects into living and not living. Is there any fuzziness in this classification?

(vii) Martin Luther King Jr. said: *"Injustice anywhere is a threat to justice everywhere."* Discuss this from the point of view of fuzzy logic.

(viii) The Telecommunications Act of 1996 replaced a communications act from 1934. It contained a provision, The Communications Decency Act which attempted to define indecent communications, i.e., pornography. How did the courts deal with this fuzzy definition? Note the fact that Congress in 1998 posted the (expensively produced) Starr Report (on the then President's sexual activities) on the internet. Large parts of this report were also reproduced in newspapers across the nation. Parts of the Starr Report were clearly indecent according to the 1996 definition of Congress. Discuss.

(ix) The Declaration of Independence states that "all men are created equal." Discuss the fuzziness in the definition of "man" and how the definition has changed incrementally from 1776 until today.

(x) Trace the "right to vote" in the United States. Is there any correlation with the definition of "man?" Reference [347] might be helpful.

(xi) Give an example of a situation where Aristotelian thinking "works" very well.

(xii) Give an example where thinking using Aristotelian logic does not "work" as well as thinking which uses fuzzy logic.

(xiii) Those who would like to see more commercial cutting of trees on America's public lands have justified this position, in part, by claiming that there are more trees on America's public lands now than there were in 1776. Assuming that this claim is true, how must the definition of a forest change if trees from one to several hundred years old are replaced by trees most of which are less than twenty to thirty years old? What are some fundamental ecological differences between these two types of forests? (For example, as early as 1990 more that 96 percent of the old growth forests in the lower 48 states had been cut.)

(xiv) Research as best you can the current uses of fuzzy logic in America, Japan and elsewhere. Why do you think eastern societies used fuzzy logic in business and engineering before it "caught on" in western societies?

(xv) The United Nations has a program, REDD, Reduced Emissions from Deforestation and Degradation, which became a topic of discussion during the Copenhagen Climate talks of December 2009, for example. The "saving of the forests" was called a breakthrough by some and greenwashing by others. What definition of "forest" was finally decided upon for the purposes of "offsetting" carbon emissions of industrial nations? For example, would clear-cutting of a diverse tropical rainforest and replacing it with the monoculture of a palm-oil plantation count as a "forest?"

(xvi) The definition of the term "murder" and its legal relatives are fairly well-defined in law. The concept does, however, involve fuzziness. For example, if operators of a coal mine consistently ignore, or dispute, hundreds of infractions called to their attention by mine inspectors (or workers brave enough to do so) declaring the mine unsafe and potentially a deadly work environment – might the mine operators be accused of some form of murder when the mine does indeed kill workers in precisely the manner predicted?

Making of Mental Models and Classical Concepts in Logic. None of us can replicate all of reality/Nature in our mental model of reality/Nature. We must simplify, discard some of the information available. This is what we do to discover patterns. The hallmark of genius is the ability to sift through the data available and concentrate on what is truly important to the subject of study – to simplify appropriately. Advice in this regard from [358] is: *"A good simplification should minimize the loss of information relevant to the problem of concern."*

Notice the word *problem.* When our models are not working, by definition we have a problem. We want or need to solve said problem. A reasonable first step is to gather information on the problem, via direct experience, indirect experience of others and literature/data searches, taking note of such details of where a particular source, if human, gains remuneration – always follow the money when money is involved.

Look for *extreme views* ardently held. There are reasons, not always explicit, for these views; there is at least a grain of truth or more than a grain to be found. *"Triangulate,"* that is, look at the problem from as many points of view, using as many sources of information as you can find and assimilate. This is sometimes referred to as doing your homework.

A Simplification Decision: Sharp or Fuzzy Logic. Among the first steps in simplification is to decide whether sharp logic is sufficient or is not sufficiently delicate to deal with the details. For example, stereotyping can be viewed as a form of Aristotelian oversimplification.[7] Thus all mathematicians are nerds. All fill in the blank are fill in the blank. In pure mathematics sharp logic is often very helpful. In social relations it rarely is.

8.3 Definitions, Assumptions and the Frame of Debate

Whenever a word is used it triggers a *frame* of mind which can be managed if you are conscious of this process. For example, in [380] we are told: "Don't think of an elephant." Of course, that is exactly what we are all thinking of when we hear that sentence! Masters of marketing, advertising, spin, propaganda, public relations and perception management surround us daily, to sell us ideas – to sell us products, wars, political candidates or the idea that professor so-and-so should be fired. None of this is new, it is just that technology has made these techniques ubiquitous and continuous, cf., Chapter 9 and VIII.

[7] Racial profiling, as in searching for criminals or terrorists, does not work. It is a form of stereotyping. For documentation of the type of police work that does apprehend law breakers see [281].

There are two useful countertechniques for dealing with the constant sales pitch. The idea of ferreting out the precise meaning of definitions, words, symbols – getting at the notions that notations represent – and continually challenging assumptions: these are two techniques developed to a fine art at least as early as the third century. Sextus Empiricus is famous as a 3^{rd} century Roman, possibly Greek, skeptic who perfected these two techniques; and who, incidentally used them to attack mathematicians of his time. I was reminded of him while watching Harold Ickes quite successfully defend himself against a hostile congressional panel – using just these techniques. What do you mean by . . . ? Are you not assuming that . . . ?

Exercise 8.3 Definitions, Assumptions and Frames

(i) Titles of bills passed by Congress are selected to frame our consciousness. How do titles such as the Patriot Act, the Healthy Forests Initiative or the Clear Skies . . . frame any discussion about them?

(ii) Take any bill that deals with terrorism, such as the Patriot Act or any such legislation passed since, and examine the definition of terrorism contained therein. Can just about any form of *peaceful* protest be considered terrorism? Are there pacifists who have been labeled as terrorists by the government? Why do you think this is?

(iii) Take any environmental piece of legislation, such as the Healthy Forest Initiative or one more recent, and discuss its actual – on the ground – effects.

(iv) Discuss the degree of fuzziness in one or more of the following sets: actions that are safe/not safe, plants that are trees/not trees, media that is pornographic/not pornographic, statements that are honest/not honest, actions that are environmentally sustainable/not sustainable, and actions that are legal/not legal.

(v) Discuss the following apparently self-contradictory statement from the point of view of fuzzy logic: "You only keep what you give away."

(vi) Discuss the following quote attributed to physicist Niels Bohr: *"A great truth is a truth whose opposite is also a great truth."* A classical book on quotations attributes this quote to Thomas Mann (1875–1955), *Essay on Freud.* (Note that to every proverb there is an equal and opposite proverb. For example, "Absence makes the heart grow fonder;" and, "Out of sight, out of mind.")

(vii) Discuss the following quote of Abraham Lincoln, (1809–1865): *"It's been my experience that folks who have no vices have very few virtues."*

(viii) H. L. Mencken was misquoted once as having said: *Every complex problem has an obvious, simple solution – which is invariably wrong.* The actual quote was: *There is always an easy solution to every human problem – neat, plausible and wrong.* Discuss what these quotes – or misquotes – have to do with fuzzy logic.

(ix) Ian McHarg, author of *Design with Nature* and [431] said: *"Brain is on trial."* Arthur C. Clarke, science fiction author, said *"It has yet to be proven that intelligence has any survival value."* Discuss what hidden assumption these quotes address.

(x) What hidden assumptions would you have to re-examine if electric power in your home and place of work (or your state) were not available for three hours? Three days? Three years? Answer the same question with petroleum substituted for electric power.

(xi) I once made the following comment to a person who did not know me very well: "I have to go now, I have a (romantic) date with a jet pilot tonight." I am a male. What possible hidden assumptions might be going through the person's mind to whom I was speaking?

(xii) Discuss how an implicit assumption that "the best science is reductionist" may promote or hold back scientific progress in some areas.

(xiii) Discuss the following comment: Whenever you communicate with someone else, you do so in a sea of intentions and hidden assumptions. Discuss how this "sea" affects the communication process.

8.4 Humans in Denial – Nature Cannot be Fooled – Gravity Exists.

The definition of *denial*: an unconscious defense mechanism characterized by a refusal to acknowledge painful realities. When one's mental model fails, denial is common. Denial in the context of interpersonal relations is common, and I will not be dealing much with this. Denial of humans in their relationship to Nature is quite another thing, and it is this type of denial that concerns me here. Whether you "believe" in evolution or not, the subject does represent volumes of empirical evidence that one cannot successfully deny exists. Whether or not your senator "believes" in global warming, there is a body of evidence that is denied at our peril. The heart of "knowing" in science is careful observation of what is going on in Nature. After centuries of observations we have discovered some patterns in Nature, and thus it is quite disavantageous to deny that laws of Nature operate. I like to say "GRAVITY EXISTS" to remind people that Nature's laws never go unenforced – whether any particular person "believes" in them or not.

Presummably any humans who were in denial about gravity tried to fly off cliffs and left no descendents. A given human may not fully appreciate Einstein's relativistic refinements of Newton's inverse square law for gravitation, but every human needs a basic understanding of gravity just to live each day. Gravity acts on us, for all practical purposes, instantly and continuously. There are less obvious laws of Nature that it is easier for humans to be in denial about – but the ultimate consequences of denial are the same.

An Example. Aerospace Professor Xinh's office was across the hall from mine. Ellison Onizuka was Dr. Xinh's graduate student. On January 28, 1986 I was talking to some students in the University Memorial Center when a friend ran up and exhaled in one breath: "The Challenger blew up! This morning on TV, they're all dead – DEAD!" It's one of those moments you never forget. Now, a short walk from my current office is a plaque with the following words:

In living memory of
ELLISON S. ONIZUKA
1946–1986
C U graduate, astronaut, and friend
who dedicated his life to the
enrichment of education and the sciences

Right after the space shuttle Challenger disaster there was a flurry of news, but not much information. Days passed. Then President Reagan appointed an investigatory commission chaired by William P. Rogers who immediately went on record saying: *"We are not going to conduct this investigation in a manner which would be unfairly critical of NASA, because we think – I certainly think – NASA has done an excellent job, and I think the American people do."* (NASA is the National Aeronautics and Space Administration.) The first of many statements in

denial; "I guess I'll never know the truth," I cursed to myself.

Then, almost by accident, a last ray of pure, honest hope. Richard P. Feynman, Nobel Prize winning physics professor from Caltech, the precocious genius on the World War II Manhattan Project, was appointed to the President's commission charged with investigating the Challenger "accident." I had the good fortune of learning much physics out of the "Feynman notes" hot off a pre-Xerox copier. I had the even better luck of meeting him some years later. I *knew* Feynman was brilliant enough to see the truth, and I guessed that *nothing* would prevent him from telling the truth that he saw. He did not disappoint.

After a burst of intense investigation Feynman zeroed in on the physical cause of the disaster: O-ring failure, due in no small measure to the freezing temperatures surrounding the launch time. In fact, Feynman did a brilliant job of cutting through to the basic truth by doing an on-national-TV-camera experiment with a clamp, a bit of O-ring and a glass of ice water, showing that at 32 degrees Fahrenheit the O-rings did not perform properly. But Feynman did not stop there. He found out that engineers knew the O-rings would most likely fail and that they had issued warnings which were ignored. Feynman went on to analyze the whole way NASA did risk assessment and management. Management in relation to several subsystems was questionable; for example, before the O-rings failed the engine turbines could have failed due to cracks/metal fatigue. Feynman found the *fundamental reason* for the Challenger loss was that NASA had gotten complacent and sloppy – that NASA was *ignoring* hard scientific *reality*. For the full and interesting story, see [188]. While most were in denial, Feynman wrote an appendix to the official report. The last sentence of his report says:

> *"For a successful technology, reality must take precedence over public relations, for Nature cannot be fooled."*

Had Feynman not been one of the twelve on the President's commission investigating the Challenger disaster, had Feynman not done a thorough, honest investigation on his own and succeeded in getting his report attached as an appendix to the "official report" of the commission, I am not sure when if ever Americans would have found out what happened.[8]

Astronaut, Dr. Kalpana Chawla, got her Ph.D. in aerospace engineering at the University of Colorado in 1988, in the same department Ellison Onizuka studied. On February 1, 2003 the space shuttle Columbia disassembled over Texas before its scheduled landing in Florida, killing all on board, including Dr. Chawla. NASA had requested satellite photos to check for damage to the Columbia while it was in orbit, before February. The U.S. government

[8]The findings of Feynman are corroborated in an NPR interview, April 29, 2001, with Chris Kraft, NASA's first flight director. Kraft was in the viewing room of Mission Control in Houston when the Challenger blew up. See also [371, 486]. For more on Feynman, cf., [229].

refused, due to its total preoccupation with Iraq.[9]

Fallacies. Classically a *fallacy* is an incorrect logical argument. There are many types of fallacy. They are categorized according to the type of logical falsehood, be it intentional or not. For example, Scott Nearing, cf., [483], lost his position in academia due in large part to a fallacy called *argumentum ad hominen*. An *ad hominen* argument attacks the person and not the argument that person is making. Nearing was labeled a Communist[10] and bounced out of academia as being "dangerous," thus diverting attention from the real reasons he was dismissed. Namely, Nearing argued unpopular views: he was anti-war and against child labor.

There are many classical books on fallacies, going back at least to Aristotle. For example, see [184, 271, 539, 76]. The ones below and others form the basis of some propaganda techniques.

Exercise 8.4 There are Many Kinds of Fallacies

(i) Can you give Aristotle's principal list of thirteen types of fallacy as he discussed them in his *Sophistical Refutations*?

(ii) Find examples of *argumentum ad temperantiam*, the moderate view is the correct one. Find examples of *argumentum ad verecundiem*, respect for authority, the advertising testimonial. Find examples of *post hoc ergo propter hoc*, after this, therefore on account of this, the fallacy of supposing that because one event follows another, then the second has been caused by the first. Find examples of *argumentum ad ignorantiam*, appeal to ignorance. Find examples of *argumentum ad antiquitam*, it is good or right because it is old. Find an example of *argumentum ad crumenam*, money is the measure of rightness; if you're so right, why ain't you rich? Find examples of *argumentum ad baculum*, arguing by intimidation. Find an example of *argumentum ad populum*, everyone believes it, so it must be true. Find an example of *argumentum ad misericordiam*, an appeal to irrelevant emotions. Find examples of *petitio principii*, circular reasoning. Find a red herring. Find an example of ex post facto statistics. Find examples of straw men, the gambler's fallacy, false precision, the undistributed middle, non sequitur, faulty generalizations, and so on – look them up. As C. L. Hamblin says in the book cited in [271]: *"We have no* theory *of fallacy at all, in the sense in which we have theories of correct reasoning or inference."*

(iii) In Chinese, accents, or more precisely tones, are part of the meaning of every word. In English they can be used slyly to mislead. I once saw a movie called *The Conversation*. The entire plot was balanced on the accent of one word. Gene Hackman was an electronic snoop, and he was recording conversations of the wife of a jealous husband. In one conversation the wife says to her lover: "He'd *kill* us if he had the chance." or was it "He'd kill *us* if he had the chance." How does emphasis change the meaning of the sentence? Find other examples of sentences wherein the entire meaning of the sentence changes if you change the word that is accented or emphasized. How can change of accent be used to mislead?

(iv) Create a new type of fallacy and see if you can fool a friend. (Then tell him/her the truth!)

(v) Discuss how racism can be exploited as a tool to prevent the formation of coalitions, e.g., divide and conquer. Coalitions might be able to solve certain problems that individual groups cannot solve separately. Can you think of any such problems? Can you think of anyone who would like to prevent the formation of certain coalitions among races, or classes?

[9] Listen to the audio archive of March 17, 2003, at www.democracynow.org.

[10] Labeling someone a Communist, end of discussion, was once the method of choice of removing persons from all walks of life, including academia, cf., [612].

8.5 The Bigger Picture Principle

Consider the following poem which artfully shows the human predicament. In this section I use the term "picture" interchangeably with "mental model."

The Blind Men and the Elephant

by John Godfrey Saxe

It was six men of Hindustan
To learning much inclined
Who went to see the elephant,
(Though all of them were blind);
That each by observation
Might satisfy his mind.

The first approached the elephant,
And happening to fall
Against his broad and sturdy side,
At once began to bawl,
"Bless me, it seems the elephant
Is very like a wall."

The second, feeling of his tusk,
Cried, "Ho! what have we here
So very round and smooth and sharp?
To me 'tis mighty clear
This wonder of an elephant
Is very like a spear."

The third approached the animal,
And happening to take
The squirming trunk within his hands,
Then boldly up and spake;
"I see," quoth he, "the elephant
Is very like a snake."

The fourth stretched out his eager hand
And felt about the knee,
"What most this mighty beast is like
Is mighty plain," quoth he;
"Tis clear enough the elephant
Is very like a tree."

The fifth who chanced to touch the ear
Said, "Even the blindest man
Can tell what this resembles most;
Deny the fact who can,
This marvel of an elephant
Is very like a fan."

The sixth no sooner had begun
About the beast to grope
Than, seizing on the swinging tail
That fell within his scope,
"I see," cried he, "the elephant
Is very like a rope."

And so these men of Hindustan
Disputed loud and long,
Each of his own opinion
Exceeding stiff and strong,
Though each was partly in the right,
And all were in the wrong!

The above poem is a metaphor for the state of humanity looking at Nature. We all have our own experiences, imaginations and hence our own mental models. We all would like to be right about our mental models, sometimes to the point of going to war.

In the poem we have six mental models and they are all consistent with and can be thought of as part of one larger model, which I call a Bigger Picture. In this case, the Bigger Picture is a model of an elephant as is commonly understood. Though difficult, if the six men got together and merged their models, subject only to consistency, a Bigger (more accurate) Picture results. This then is the essence of my Bigger Picture Principle. Recall the Axiom of Consistency, cf., Section 7.2, which asserts that Nature (Reality) is consistent.[11]

The Bigger Picture Principle. *Suppose there are given two or more mental models of some aspect of Nature. Merge these models into one model subject to the requirement that this new model is consistent.*[12] *Then this merged model,*

[11]The sharp logical concept of consistency means free from contradiction, A and not A cannot happen. Since I insist on allowing the option of fuzzy logic, statements that are say .1 A and .9 not A are allowed. What is not allowed is a statement that is 1.0 A and 1.0 not A. A statement that is .5 A and .5 not A is allowed. If you find all this confusing I suggest you read [365, 358, 596] or other books on fuzzy logic. I can cheat here and say that any model is consistent if it agrees with (observations of) Nature.

[12]I can take as a working definition of consistent: "agrees with all observations of and experiments in Nature." Thus the axiom of consistency is a bit of circular reasoning, or a

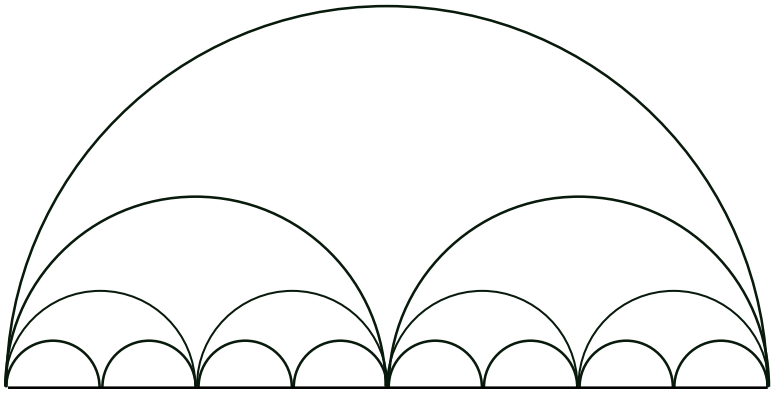

FIGURE 8.3: Two Curves Close in Distance but Not in Length

called a Bigger Picture, is more likely to be a more accurate model of the aspect of reality in question than each smaller model taken separately. (See also Section 1.4.)

The more you can explain with your model the better it is, by definition.

Examples of Bigger Pictures. What Does It Mean for Two Curves to Be Close? Consider Figure 8.3. The length of the straight line segment from 0 to 1 shall be taken to be 1 (unit). The length of the semicircle whose base is this same segment is $\frac{\pi}{2}$, since the circumference of a circle is given by the product of its diameter times π. (Recall that $\pi \approx 3.141592654$.) The length of the curve consisting of two semicircles is also $\frac{\pi}{2}$. Can you see this?[13] The length of the curve consisting of four semicircles is also $\frac{\pi}{2}$. Can you see this?[14] Perhaps you now see a pattern, and a possible problem with your intuition! There is a sequence of curves each of length $\frac{\pi}{2}$ converging uniformly to the line segment of length 1.

Your intuition might be telling you that if curves are "close" to each other, then their lengths should be close to the same number. This evidently is not true. Your picture, or mental model of the situation needs an overhaul. See if you can resolve this paradox by yourself by making your picture/mental model bigger and more at one with what is (before looking further).

tautology. Nature defines consistency. Thus any model, newer, bigger or otherwise, should undergo a reality check. Does it agree with Nature? A deeper discussion I leave to the reader to investigate as an exercise.

[13]You have two semicircles, each with a diameter of $\frac{1}{2}$.

[14]This is because you have four semicircles each with a diameter of $\frac{1}{4}$.

Exercise 8.5 A Bigger Picture Can Entail More Refined Definitions
Give yourself another day to contemplate the situation in Figure 8.3 before reading on. If you want to believe that curves that are "close" have lengths that are "close" you will have to be careful and make a precise definition of what it means for one curve to be close to another. If the points along one curve are uniformly close to the points of another, this is intuitively necessary – but not sufficient – for the two curves to be "close." Can you come up with a definition of "closeness" of curves which when applied to Figure 8.3 shows that as more and more semicircles are introduced the resulting curve moves farther and farther away from the straight line segment of length 1?[15]

The purpose of the above brain teaser is to give an explicit example of past mathematical research that creates a bigger more honest picture than existed before, and hence deepens understanding. There are many other examples, like Feynman integrals, which have defied a complete understanding for decades. Mathematical research is indeed very active today, making bigger more honest pictures.

Exercise 8.6 Counting Infinitely Many Points with Geometry
(i) Can you make an argument that a circle with one point removed has the same number of points as a straight line which is infinitely long?
(ii) Can you make an argument, similar to the argument in (i), that a sphere with one point removed has the same number of points as a flat plane infinite in all directions?

For an illustration of the Bigger Picture Principle involving the CIA see Section 23.4.

Exercise 8.7 A Bigger Picture of Climate Change
Critics of climate change science often make comments to the effect that last year wasn't so warm; or, the sun is getting warmer; or, climate scientists have an agenda in co-operation with proponents of world government; and so on. Can the Bigger Picture Principle be used in this context to incorporate and explain all the objections that the sceptics of global warming bring up?
(i) How many of the sceptics' objections are actually purposeful misprepresentations, cf., [317]?
(ii) Which sceptics' objections actually bring up true scientific phenonmena, but the effects of these phenomena are not quantitatively significant?

[15] Hint: The astute observer will notice that the curves consisting of semicircles have more cusps (points at the end of a semicircle) the more semicircles there are. Thus such a curve is "far away" from the straight line segment from 0 to 1 in the sense that the tangent line to the curve (made up of semicircles) at each cusp is perpendicular to, i.e., orthogonal to, makes a ninety degree angle with, the corresponding tangent to the line segment from 0 to 1. (The line segment joining 0 to 1 is a flat curve, and the tangent to any point on this curve is the line through that point parallel to the line segment from 0 to 1. In other words, the line segment is essentially its own tangent at every point of itself.) Mathematicians can say that two curves are *close* if the points of one curve are uniformly close to the points of the other *and* the tangent at a point on one curve is close to the tangent at the corresponding point of the other curve. (Close tangents have slopes that are close, i.e., they are nearly parallel.) This definition can be made a great deal more precise by using calculus, and if you are interested there is a long discussion of the details in [379, pp. 325–334]. This book should be quite interesting to anyone who wants to learn or teach mathematics, or who is just interested in "where it comes from." Also of interest is a review of this book in the November 2001 issue of the Notices of the American Mathematical Society, pp. 1182–1188.

(iii) Has anyone proposed an effect that would quantitatively negate the effects of Arrhenius's CO_2 Greenhouse Gas Law, cf. Section 1.9?

(iv) Have any sceptics addressed the acidification of the world's oceans?

(v) How "Big" is the total picture of all climate change sceptics taken together?

(vi) Have the IPCC model's already addressed climate-change sceptic's objections? To what extent is climate-change scepticism just a form of denial?