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THE PURSUIT OF BEAUTY

Yitang Zhang solves a pure-math mystery.

BY ALEC WILKINSON

Unable to get an academic position, Zhang kept the books for a Subway franchise.

PHOTOGRAPH BY PETER BOHLER

I don't see what difference it can make now to reveal that I passed high-school math only because I cheated. I could add and subtract and multiply and divide, but I entered the wilderness when words became equations and x's and y's. On test days, I sat next to Bob Isner or Bruce Gelfand or Ted Chapman or Donny Chamberlain—smart boys whose handwriting I could read—and divided my attention between his desk and the teacher's eyes. Having skipped me, the talent for math concentrated extravagantly in one of my nieces, Amie Wilkinson, a professor at the University of Chicago. From Amie I first heard about Yitang Zhang, a solitary, part-time calculus teacher at the University of New Hampshire who received several prizes, including a MacArthur award in September, for solving a problem that had been open for more than a hundred and fifty years.

The problem that Zhang chose, in 2010, is from number theory, a branch of pure mathematics. Pure mathematics, as opposed to applied mathematics, is done with no practical purposes in mind. It is as close to art and philosophy as it is to engineering. "My result is useless for industry," Zhang said. The British mathematician G. H. Hardy wrote in 1940 that mathematics is, of "all the arts and sciences, the most austere and the most remote." Bertrand Russell called it a refuge from "the dreary exile of the actual world." Hardy believed emphatically in the precise aesthetics of math. A mathematical proof, such as Zhang produced, "should resemble a simple and clear-cut constellation," he wrote, "not a scattered cluster in the Milky Way." Edward Frenkel, a math professor at the University of California, Berkeley, says Zhang's proof has "a renaissance beauty," meaning that though it is deeply complex, its outlines are easily apprehended. The pursuit of beauty in pure mathematics is a tenet. Last year, neuroscientists in Great Britain discovered that the same part of the brain that is activated by art and music was activated in the brains of mathematicians when they looked at math they regarded as beautiful.

Zhang's problem is often called "bound gaps." It concerns prime numbers—those which can be divided cleanly only by one and by themselves: two, three, five, seven, and so on—and the question of whether there is a boundary within which, on an infinite number of occasions, two consecutive prime numbers can be found, especially out in the region where the numbers are so large that it would take a book to print a single one of them. Daniel Goldston, a professor at San Jose State University; János Pintz, a fellow at the Alfréd Rényi Institute of Mathematics, in Budapest; and Cem Yıldırım, of Boğaziçi University, in Istanbul, working together in 2005, had come closer than anyone else to establishing whether there might be a boundary, and what it might be. Goldston didn't think he'd see the answer in his lifetime. "I thought it was impossible," he told me.

Zhang, who also calls himself Tom, had published only one paper, to quiet acclaim, in 2001. In 2010, he was fifty-five. "No mathematician should ever allow himself to forget that mathematics, more than any other art or science, is a young man's game," Hardy wrote. He also wrote, "I do not know of an instance of a major mathematical advance initiated by a man past fifty." Zhang had received a Ph.D. in algebraic geometry from Purdue in 1991. His adviser, T. T. Moh, with whom he parted unhappily, recently wrote a description on his Web site of Zhang as a graduate student: "When I looked into his eyes, I found a disturbing soul, a burning bush, an explorer who wanted to reach the North Pole." Zhang left Purdue without Moh's support, and, having published no papers, was unable to find an academic job. He lived, sometimes with friends, in Lexington, Kentucky, where he had occasional work, and in New York City, where he also had friends and occasional work. In Kentucky, he became involved with a group interested in Chinese democracy. Its slogan was "Freedom, Democracy, Rule of Law, and Pluralism." A member of the group, a chemist in a lab, opened a Subway franchise as a means of raising money. "Since Tom was a genius at numbers," another member of the group told me, "he was invited to help him." Zhang kept the books. "Sometimes, if it was busy at the store, I helped with the cash register," Zhang told me recently. "Even I knew how to make the sandwiches, but I didn't do it so much." When Zhang wasn't working, he would go to the library at the University of Kentucky and read journals in algebraic geometry and number theory. "For years, I didn't really keep up my dream in mathematics," he said.

"You must have been unhappy."

He shrugged. "My life is not always easy," he said.

With a friend's help, Zhang eventually got his position in New Hampshire, in 1999. Having chosen bound gaps in 2010, he was uncertain of how to find a way into the problem. "I am thinking, Where is the door?" Zhang said. "In the history of this problem, many mathematicians believed that there should be a door, but they couldn't find it. I tried several doors. Then I start to worry a little that there is no door."

"Were you ever frustrated?"

"I was tired," he said. "But many times I just feel peaceful. I like to walk and think. This is my way. My wife would see me and say, 'What are you doing?' I said, 'I'm working, I'm thinking.' She didn't understand. She said, 'What do you mean?' " The problem was so complicated, he said, that "I had no way to tell her."

According to Deane Yang, a professor of mathematics at the New York University Polytechnic School of Engineering, a mathematician at the beginning of a difficult problem is "trying to maneuver his way into a maze. When you try to prove a theorem, you can almost be totally lost to knowing exactly where you want to go. Often, when you find your way, it happens in a moment, then you live to do it again."

Zhang is deeply reticent, and his manner is formal and elaborately polite. Recently, when we were walking, he said, "May I use these?" He meant a pair of clip-on shades, which he held toward me as if I might want to examine them first. His enthusiasm for answering questions about himself and his work is slight. About half an hour after I had met him for the first time, he said, "I have a question." We had been talking about his childhood. He said, "How many more questions you going to have?" He depends heavily on three responses: "Maybe," "Not so much," and "Maybe not so much." From diffidence, he often says "we" instead of "I," as in, "We may not think this approach is so important." Occasionally, preparing to speak, he hums. After he published his result, he was invited to spend six months at the Institute for Advanced Study, in Princeton. The filmmaker

George Csicsery has made a documentary about Zhang, called "Counting from Infinity," for the Mathematical Sciences Research Institute, in Berkeley, California. In it, Peter Sarnak, a member of the Institute for Advanced Study, says that one day he ran into Zhang and said hello, and Zhang said hello, then Zhang said that it was the first word he'd spoken to anyone in ten days. Sarnak thought that was excessive, even for a mathematician, and he invited Zhang to have lunch once a week.

Matthew Emerton, a professor of math at the University of Chicago, also met Zhang at Princeton. "I wouldn't say he was a standard person," Emerton told me. "He wasn't gregarious. I got the impression of him being reasonably internal. He had received another prize, so the people around him were talking about that. Probably most mathematicians are very low-key about getting a prize, because you're not in it for the prize, but he seemed particularly low-key. It didn't seem to affect him at all." Deane Yang attended three lectures that Zhang gave at Columbia in 2013. "You expect a guy like that to want to show off or explain how smart he is," Yang said. "He gave beautiful lectures, where he wasn't trying to show off at all." The first talk that Zhang gave on his result was at Harvard, before the result was published. A professor there, Shing-Tung Yau, heard about Zhang's paper, and invited him. About fifty people showed up. One of them, a Harvard math professor, thought Zhang's talk was "pretty incomprehensible." He added, "The problem is that this stuff is hard to talk about, because everything hinges on some delicate technical understandings." Another Harvard professor, Barry Mazur, told me that he was "moved by his intensity and how brave and independent he seemed to be."

In New Hampshire, Zhang works in an office on the third floor of the math and computer-science building. His office has a desk, a computer, two chairs, a whiteboard, and some bookshelves. Through a window he looks into the branches of an oak tree. The books on his shelves have titles such as "An Introduction to Hilbert Space" and "Elliptic Curves, Modular Forms, and Fermat's Last Theorem." There are also books on modern history and on Napoleon, who fascinates him, and copies of Shakespeare, which he reads in Chinese, because it's easier than Elizabethan English.

Eric Grinberg, the chairman of the math department at the University of Massachusetts Boston, was a colleague of Zhang's in New Hampshire from 2003 to 2010. "Tom was very modest, very unassuming, never asked for anything," Grinberg told me. "We knew he was working on something important. He uses paper and a pencil, but the only copy was on his computer, and about once a month I would go in and ask, 'Do you mind if I make a backup?' Of course, it's all in his head anyway. He's above average in that."

Zhang's memory is abnormally retentive. A friend of his named Jacob Chi said, "I take him to a party sometimes. He doesn't talk, he's absorbing everybody. I say, 'There's a human decency; you must talk to people, please.' He says, 'I enjoy your conversation.' Six months later, he can say who sat where and who started a conversation, and he can repeat what they said."

"I may think socializing is a way to waste time," Zhang says. "Also, maybe I'm a little shy."

A few years ago, Zhang sold his car, because he didn't really use it. He rents an apartment about four miles from campus and rides to and from his office with students on a school shuttle. He says that he sits on the bus and thinks. Seven days a week, he arrives at his office around eight or nine and stays until six or seven. The longest he has taken off from thinking is two weeks. Sometimes he wakes in the morning thinking of a math problem he had been considering when he fell asleep. Outside his office is a long corridor that he likes to walk up and down. Otherwise, he walks outside.

Zhang met his wife, to whom he has been married for twelve years, at a Chinese restaurant on Long Island, where she was a waitress. Her name is Yaling, but she calls herself Helen. A friend who knew them both took Zhang to the restaurant and pointed her out. "He asked, 'What do you think of this girl?' " Zhang said. Meanwhile, she was considering him. To court her, Zhang went to New York every weekend for several months. The following summer, she came to New Hampshire. She didn't like the winters, though, and moved to California, where she works at a beauty salon. She and Zhang have a house in San Jose, and he spends school vacations there.

Until Zhang was promoted to professor, last year, as a consequence of his proof, his appointment had been tenuous. "I was chair of the math department, and I had to go to him from time to time and remind him this was not a permanent position," Eric Grinberg said. "We were grateful to him, but it's not guaranteed. He always said that he very much appreciated the time he had spent in New Hampshire."

Zhang devoted himself to bound gaps for a couple of years without finding a door. "We couldn't see any hope," he said. Then, on July 3, 2012, in the middle of the afternoon, "within five or ten minutes, the way is open."

Zhang was in Pueblo, Colorado, visiting his friend Jacob Chi, who is a music professor at Colorado State University-Pueblo. A few months earlier, Chi had reminded Zhang that he had promised one day to teach his son, Julius, calculus, and since Julius was about to be a senior in high school Chi had called and asked, "Do you keep your promise?" Zhang spent a month at the Chis'. Each morning, he and Julius worked for about an hour. "He didn't have a set curriculum," Julius told me. "It all just flowed from his memory. He mentioned once that he didn't have any numbers in his phone book. He memorized them all."

Zhang had planned a break from work in Colorado, and hadn't brought any notes with him. On July 3rd, he was walking around the Chis' back yard. "We live in the mountains, and the deer come out, and he was smoking a cigarette and watching for the deer," Chi said. "No deer came," Zhang said. "Just walking and thinking, this is my way." For about half an hour, he walked around at a loss.

In "The Psychology of Invention in the Mathematical Field," published in 1945, Jacques Hadamard quotes a mathematician who says, "It often seems to me, especially when I am alone, that I find myself in another world. Ideas of numbers seem to live. Suddenly, questions of any kind rise before my eyes with their answers." In the back yard, Zhang had a similar experience. "I see numbers, equations, and something even—it's hard to say what it is," Zhang said. "Something very special. Maybe numbers, maybe equations—a mystery, maybe a vision. I knew that, even though there were many details to fill in, we should have a proof. Then I went back to the house."

Zhang didn't say anything to Chi about his breakthrough. That evening, Chi was conducting a dress rehearsal for a Fourth of July concert in Pueblo, and Zhang went with him. "After the concert, he couldn't stop humming 'The Stars and Stripes Forever,' " Chi said. "All he would say was 'What a great song.' "

I asked Zhang, "Are you very smart?" and he said, "Maybe, a little." He was born in Shanghai in 1955. His mother was a secretary in a government office, and his father was a college professor whose field was electrical engineering. As a small boy, he began "trying to know everything in mathematics," he said. "I became very thirsty for math." His parents moved to Beijing for work, and Zhang remained in Shanghai with his grandmother. The revolution had closed the schools. He spent most of his time reading math books that he ordered from a bookstore for less than a dollar. He was fond of a series whose title he translates as "A Hundred Thousand Questions Why." There were volumes for physics, chemistry, biology, and math. When he didn't understand something, he said, "I tried to solve the problem myself, because no one could help me."

Zhang moved to Beijing when he was thirteen, and when he was fifteen he was sent with his mother to the countryside, to a farm, where they grew vegetables. His father was sent to a farm in another part of the country. If Zhang was seen reading books on the farm, he was told to stop. "People did not think that math was important to the class struggle," he said. After a few years, he returned to Beijing, where he got a job in a factory making locks. He began studying to take the entrance exam for Peking University, China's most respected school: "I spent several months to learn all the high-school physics and chemistry, and several to learn history. It was a little hurried." He was admitted when he was twenty-three. "The first year, we studied calculus and linear algebra—it was very exciting," Zhang said. "In the last year, I selected number theory as my specialty." Zhang's professor insisted, though, that he change his major to algebraic geometry, his own field. "I studied it, but I didn't really like it," Zhang said. "That time in China, still the idea was like this: the individual has to follow the interest of the whole group, the country. He thought algebraic geometry was more important than number theory. He forced me. He was the university president, so he had the authority."

During the summer of 1984, T. T. Moh visited Peking University from Purdue and invited Zhang and several other students, recommended to him by Chinese professors, to do graduate work in his department. One of Moh's specialties is the Jacobian conjecture, and Zhang was eager to work on it. The Jacobian conjecture, a problem in algebraic geometry that was introduced in 1939 and is still unsolved, stipulates certain simple conditions that, if satisfied, enable someone to solve a series of complicated equations. It is acknowledged as being beyond the capacities of a graduate student and approachable by only the most accomplished algebraic geometrists. A mathematician described it to me as a "disaster problem," for the trouble it has caused. For his thesis, Zhang submitted a weak form of the conjecture, meaning that he attempted to prove something implied by the conjecture, rather than to prove the conjecture itself.

After Zhang received his doctorate, he told Moh that he was returning to number theory. "I was not the happiest," Moh wrote me. "However, I was for the student's right to change fields, so I kept my smile and said bye to him. For the past 22 years, I knew nothing about him."

After graduating, most of the Chinese students went into either computer science or finance. One of them, Perry Tang, who had known Zhang in China, took a job at Intel. In 1999, he called Zhang. "I thought it was unfair for him not to have a professional job," Tang said. He and Zhang had a classmate at Peking University who had become a professor of math at the University of New Hampshire, and when the friend said that he was looking for someone to teach calculus Tang recommended Zhang. "He decided to try him at a temporary position," Tang said.

Zhang finished "Bounded Gaps Between Primes" in late 2012; then he spent a few months methodically checking each step, which he said was "very boring." On April 17, 2013, without telling anyone, he sent the paper to *Annals of Mathematics*, widely regarded as the profession's most prestigious journal. In the *Annals* archives are unpublished papers claiming to have solved practically every math problem that anyone has ever thought of, and others that don't really exist. Some are from people who "know a lot of math, then they go insane," a mathematician told me. Such people often claim that everyone else who has attacked the problem is wrong. Or they announce that they have solved several problems at once, or "they say they have solved a famous problem along with some unified-field theory in physics," the mathematician said. Journals such as *Annals* are always skeptical of work from someone they have never heard of.

In 2013, *Annals* received nine hundred and fifteen papers and accepted thirty-seven. The wait between acceptance and publication is typically around a year. When a paper arrives, "it is read quickly, for worthiness," Nicholas Katz, the Princeton professor who is the journal's editor, told me, and then there is a deep reading that can take months. "The paper I can't evaluate off the top of my head, my role is to know whom to ask," Katz said. "In this case, the person wrote back pretty quickly to say, 'If this is correct, it's really fantastic. But you should be careful. This guy posted a paper once, and it was wrong. He never published it, but he didn't take it down, either.' " The reader meant a paper that Zhang posted on the Web site arxiv.org, where mathematicians often post results before submitting them to a journal, in order to have them seen quickly. Zhang posted a paper in 2007 that fell short of a proof. It concerned another famous problem, the Landau-Siegel zeros conjecture, and he left it up because he hopes to correct it.

Katz sent "Bounded Gaps Between Primes" to a pair of readers, who are called referees. One of them was Henryk Iwaniec, a professor at Rutgers, whose work was among that which Zhang had drawn on. "I glanced for a few minutes," Iwaniec told me. "My first impression was: So many claims have become wrong. And I thought, I have other work to do. Maybe I'll postpone it. Remember that he was an unknown guy. Then I got a phone call from a friend, and it happened he was also reading the paper. We were going to be together for a week at the Institute for Advanced Study, and the intention was to do other work, but we were interrupted with this paper to read."

Iwaniec and his friend, John Friedlander, a professor at the University of Toronto, read with increasing attention. "In these cases, you don't read A to Z," Iwaniec said. "You look first at where is the idea. There had been nothing written on the subject since 2005. The problem was too difficult to solve. As we read more and more, the chance that the work was correct was becoming really great. Maybe two days later, we started looking for completeness, for connections. A few days passed, we're checking line by line. The job is no longer to say the work is fine. We are looking to see if the paper is truly correct."

After a few weeks, Iwaniec and Friedlander wrote to Katz, "We have completed our study of the paper 'Bounded Gaps Between Primes' by Yitang Zhang." They went on, "The main results are of the first rank. The author has succeeded to prove a landmark theorem in the distribution of prime numbers." And, "Although we studied the arguments very thoroughly, we found it very difficult to spot even the smallest slip. . . . We are very happy to strongly recommend acceptance of the paper for publication in the *Annals*."

Once Zhang heard from *Annals*, he called his wife in San Jose. "I say, 'Pay attention to the media and newspapers,' " he said. " 'You may see my name,' and she said, 'Are you drunk?' "

No formula predicts the occurrence of primes—they behave as if they appear randomly. Euclid proved, in 300 B.C., that there is an infinite number of primes. If you imagine a line of all the numbers there are, with ordinary numbers in green and prime numbers in red, there are many red numbers at the beginning of the line: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47 are the primes below fifty. There are twenty-five primes between one and a hundred; 168 between one and a thousand; and 78,498 between one and a million. As the primes get larger, they grow scarcer and the distances between them, the gaps, grow wider.

"I hear this place is a hotbed of international pancakes."

Prime numbers have so many novel qualities, and are so enigmatic, that mathematicians have grown fetishistic about them. Twin primes are two apart. Cousin primes are four apart, sexy primes are six apart, and neighbor primes are adjacent at some greater remove. From "Prime Curios!," by Chris Caldwell and G. L. Honaker, Jr., I know that an absolute prime is prime regardless of how its digits are arranged: 199; 919; 991. A beastly prime has 666 in the center. The number 700666007 is a beastly palindromic prime, since it reads the same forward and backward. A circular prime is prime through all its cycles or formulations: 1193, 1931, 9311, 3119. There are Cuban primes, Cullen primes, and curved-digit primes, which have only curved numerals—0, 6, 8, and 9. A prime from which you can remove numbers and still have a prime is a deletable prime, such as 1987. An emirp is prime even when you reverse it: 389, 983. Gigantic primes have more than ten thousand digits, and holey primes have only digits with holes (0, 4, 6, 8, and 9). There are Mersenne primes; minimal primes; naughty primes, which are made mostly from zeros (naughts); ordinary primes; Pierpont primes; plateau primes, which have the same interior numbers and smaller numbers on the ends, such as 1777771; snowball primes, which are prime even if you haven't finished writing all the digits, like 73939133; Titanic primes; Wagstaff primes; Wall-Sun-Sun primes; Wolstenholme primes; Woodall primes; and Yarborough primes, which have neither a 0 nor a 1.

"Bounded Gaps Between Primes" is a back-door attack on the twin-prime conjecture, which was proposed in the nineteenth century and says that, no matter how far you travel on the number line, even as the gap widens between primes you will always encounter a pair of primes that are separated by two. The twin-prime conjecture is still unsolved. Euclid's proof established that there will always be primes, but it says nothing about how far apart any two might be. Zhang established that there is a distance within which, on an infinite number of occasions, there will always be two primes.

"You have to imagine this coming from nothing," Eric Grinberg said. "We simply didn't know. It is like thinking that the universe is infinite, unbounded, and finding it has an end somewhere." Picture it as a ruler that might be applied to the line of green and red numbers. Zhang chose a ruler of a length of seventy million, because a number that large made it easier to prove his conjecture. (If he had been able to prove the twin-prime conjecture, the number for the ruler would have been two.) This ruler can be moved along the line of numbers and enclose two primes an infinite number of times. Something that holds for infinitely many numbers does not necessarily hold for all. For example, an infinite number of numbers are even, but an infinite number of numbers are not even, because they are odd. Similarly, this ruler can also be moved along the line of numbers an infinite number of times and not enclose two primes.

From Zhang's result, a deduction can be made, which is that there is a number smaller than seventy million which precisely defines a gap separating an infinite number of pairs of primes. You deduce this, a mathematician told me, by means of the pigeonhole principle. You have an infinite number of pigeons, which are pairs of primes, and you have seventy million holes. There is a hole for primes separated by two, by three, and so on. Each pigeon goes in a hole. Eventually, one hole will have an infinite number of pigeons. It isn't possible to know which one. There may even be many, there may be seventy million, but at least one hole will have an infinite number of pigeons.

Having discovered that there is a gap, Zhang wasn't interested in finding the smallest number defining the gap. This was work that he regarded as a mere technical problem, a type of manual labor —"ambulance chasing" is what a prominent mathematician called it. Nevertheless, within a week of Zhang's announcement mathematicians around the world began competing to find the lowest number. One of the observers of their activity was Terence Tao, a professor at U.C.L.A. Tao had the idea for a coöperative project in which mathematicians would work to lower the number rather than "fighting to snatch the lead," he told me.

The project, called Polymath8, started in March of 2013 and continued for about a year. Incrementally, relying also on work by a young British mathematician named James Maynard, the participants reduced the bound to two hundred and forty-six. "There are several problems with going lower," Tao said. "More and more computer power is required—someone had a high-powered computer running for two weeks to get that calculation. There were also theoretical problems. With current methods, we can never get better than six, because of something called the parity problem, which no one knows how to get past." The parity problem says that primes with certain behaviors can't be detected with current methods. "We never strongly believed we would get to two and prove the twin-prime conjecture, but it was a fun journey," Tao said.

"Is there a talent a mathematician should have?"

"Concentration," Zhang said. We were walking across the campus in a light rain. "Also, you should never give up in your personality," he continued. "Maybe something in front of you is very complicated, it's lengthy, but you should be able to pick up the major points by your intuition."

When we reached Zhang's office, I asked how he had found the door into the problem. On a whiteboard, he wrote, "Goldston-Pintz-Yıldırım"and "Bombieri-Friedlander-Iwaniec." He said, "The first paper is on bound gaps, and the second is on the distribution of primes in arithmetic progressions. I compare these two together, plus my own innovations, based on the years of reading in the library."

When I asked Peter Sarnak how Zhang had arrived at his result, he said, "What he did was look way out of reach. Maybe forty years ago the problem appeared hopeless, but in 2005 Goldston-Pintz-Yıldırım put it at the doorstep. Everybody thought, Now we're very close, but by 2011 no one was making any progress. Bombieri, Friedlander, and Iwaniec had the other important work, but it looked like you couldn't combine their ideas with Goldston. Their work was just not flexible enough to jive—it insisted on some side conditions. Then Zhang comes along. A lot of people use theorems like a computer. They think, If it is correct, then good, I'll use it. You couldn't use the Bombieri-Friedlander-Iwaniec, though, because it wasn't flexible enough. You have to take my word, because even to a serious mathematician this would be difficult to explain. Zhang understood the techniques deeply enough so as to be able to modify Bombieri-Friedlander-Iwaniec and cross this bridge. This is the most significant thing about what he has done mathematically. He's made the Bombieri-Friedlander-Iwaniec technique about the distribution of prime numbers a tool for any kind of study of primes. A development that began in the eighteen-hundreds continued through him."

“Our conditions needed to be relaxed,” Iwaniec told me. “We tried, but we couldn’t remove them. We didn’t try long, because after failing you just start thinking there is some kind of natural barrier, so we gave up.”

I asked if he was surprised by Zhang’s result. “What Zhang did was sensational,” he said. “His work is a masterpiece. When you talk of number theory, a lot of the beauty is the machinery. Zhang somehow completely understood the situation, even though he was working alone. That’s how he surprised. He just amazingly pushed further some of the arguments in these papers.”

Zhang used a very complicated form of a simple device for finding primes called a sieve, invented by a Greek named Eratosthenes, a contemporary of Archimedes. To use a simple sieve to find the primes less than a thousand, say, you write down all the numbers, then cross out the multiples of two, which can’t be prime, since they are even. Then you cross out the multiples of three, then five, and so on. You have to go only as far as the multiples of thirty-one. Zhang used a different sieve from the one that others had used. The previous sieve excluded numbers once they grew too far apart. With it, Goldston, Pintz, and Yıldırım had proved that there were always two primes separated by something less than the average distance between primes that large. What they couldn’t identify was a precise gap. Zhang succeeded partly by making the sieve less selective.

I asked Zhang if he was working on something new. “Maybe two or three problems I would like to solve,” he said. “Bound gaps is successful, but still I have something else.”

“Will it be as important?”

“Yes.”

According to other mathematicians, Zhang is working on his incomplete result for the Landau-Siegel zeros conjecture. “If he succeeds, it would be much more dramatic,” Peter Sarnak said. “We don’t know how close he is, but he’s proved that he’s a genius. There’s no question about that. He’s also proved that he can speak with something over many years. Based on that, his chances are not zero. They’re positive.”

“Many people have tried that problem,” Iwaniec said. “He’s a private guy. Nothing is rushed. If it takes him another ten years, that’s fine with him. Unless you tackle a problem that’s already solved, which is boring, or one whose solution is clear from the beginning, mostly you are stuck. But Zhang is willing to be stuck much longer.”

Zhang’s preference for undertaking only ambitious problems is rare. The pursuit of tenure requires an academic to publish frequently, which often means refining one’s work within a field, a task that Zhang has no inclination for. He does not appear to be competitive with other mathematicians, or resentful about having been simply a teacher for years while everyone else was a professor. No one who knows him thinks that he is suited to a tenure-track position. “I think what he did was brilliant,” Deane Yang told me. “If you become a good calculus teacher, a school can become very dependent on you. You’re cheap and reliable, and there’s no reason to fire you. After you’ve done that a couple of years, you can do it on autopilot; you have a lot of free time to think, so long as you’re willing to live modestly. There are people who try to work nontenure jobs, of course, but usually they’re nuts and have very dysfunctional personalities and lives, and are unpleasant to deal with, because they feel disrespected. Clearly, Zhang never felt that.”

One day, I arrived at Zhang's office as he was making tea. There was a piece of paper on his desk with equations on it and a pen on top of the paper. Zhang had an envelope in one hand. "I had a letter from an old friend," he said. "We have been separated for many years, and now he found me."

He took a pair of scissors from a drawer and cut open the envelope so slowly that he seemed to be performing a ritual. The letter was written in Chinese characters. He sat on the edge of his chair and read slowly. He put the letter down and took from the envelope a photograph of a man and a woman and a child on a sofa with a curtain in the background. He returned to reading the letter and then he put it back in the envelope and in the drawer and closed the drawer. "His new address is in Queens," he said. Then he picked up his tea and blew on it and faced me, looking at me over the top of the cup like someone peering over a wall.

I asked about Hardy's observations regarding age—Hardy also wrote, "A mathematician may still be competent enough at sixty, but it is useless to expect him to have original ideas."

"This may not apply to me," Zhang said. He put his tea on the desk and looked out the window. "Still I think I have intuition," he said. "Still I am confident of myself. Still I have some other visions." ♦

* (http://#correctionasterisk)An earlier version of this article misstated the name of the documentary about Zhang.

Alec Wilkinson has published ten books, including "The Protest Singer" and "The Ice Balloon."