Prop 2.2 p. 7 of Bordemann-Wagemann

Let $\varphi : (M, \circ, 1) \longrightarrow (N, \circ, 1)$ be a morphism of lie racks so $\varphi : M \to N$ $\varphi(1) = 1$

$$\varphi(x \circ y) = \varphi(x) \circ \varphi(y)$$

Then $\varphi'(1) : T_1M \to T_1N$ is a morphism of leibniz algebras

i.e. $\varphi'(1)[\xi, \eta] = [\varphi'(1)\xi, \varphi'(1)\eta]$

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proof:

$$y \longrightarrow \phi(x)y = x \circ y \longrightarrow \varphi(x \circ y)$$

let $f(y) = \varphi \circ \phi(x)$

$= \varphi(x \circ y)$

$$M \xrightarrow{\phi(x)} M \xrightarrow{\varphi} N$$

$$T_1M \longrightarrow T_1M \longrightarrow T_1N$$

$$y \longrightarrow \varphi(y) \longrightarrow \varphi(x) \circ \varphi(y)$$

Let $g(y) = \phi(\varphi(x)) \circ \varphi$

$= \varphi(x) \circ \varphi(y)$

$$M \xrightarrow{\varphi} N \xrightarrow{\phi(\varphi(x))} N$$

$f = g$ so $f'(y) = g'(y)$

$$f'(y) = \varphi'(\phi(x)y) \circ \phi(x)'(y)$$

$$f'(1) = \varphi'(1) \circ \varphi(x)'(1) \quad : T_1 M \to T_1 N$$

$$= \varphi'(1) \circ \Phi(x)$$

$$f'(1)(\xi) = \varphi'(1)(\Phi(x)\xi) \qquad \boxed{f'(1) = \varphi'(1) \circ \Phi(x)}$$

$$g'(y) = \phi(\varphi(x))'(\varphi(y)) \circ \varphi'(y)$$

$$g'(1) = \phi(\varphi(x))'(1) \circ \varphi'(1)$$

$$g'(1)(\xi) = \phi(\varphi(x))'(1)(\varphi'(1)\xi)$$

$$= \Phi(\varphi(x))(\varphi'(1)\xi) \qquad \boxed{g'(1) = \Phi(\varphi(x)) \circ \varphi'(1)}$$

$h(x) = f'(1)$ and $h(x) = g'(1) = R_{\varphi'(1)} \circ \Phi \circ \varphi$

$$= \varphi'(1) \circ \Phi(x)$$

$h'(x) = \varphi'(1) \circ \Phi'(x)$ $\qquad$ $h'(x) = R'_{\varphi'(1)}(\Phi(\varphi(x))) \circ \Phi'(\varphi(x)) \circ \varphi'(x)$

$h'(1) = \varphi'(1) \circ \Phi'(1)$ $\qquad$ $h'(1) = R_{\varphi'(1)} \circ \mathrm{ad} \circ \varphi'(1)$

$= \varphi'(1) \circ \mathrm{ad}$ $\qquad$ $h'(1)(\xi) = \mathrm{ad}(\varphi'(1)\xi) \circ \varphi'(1)$

$h'(1)(\xi) = \varphi'(1)(\mathrm{ad}\,\xi)$ $\qquad$ $h'(1)(\xi).\eta = [\varphi'(1)\xi, \varphi'(1)\eta]$

$h'(1)(\xi).\eta = \varphi'(1)[\xi, \eta]$

Q.E.D.