Filiform & Null-filiform Leibniz algebras

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In reference [55] Definition 2.2

let L be a Leibniz algebra

$L^1 = L,\ L^2 = [L^1, L], \ldots L^{k+1} = [L^k, L]$

If $n = \dim L$ then L is <u>filiform</u>

if $\dim L^i = n - i$, $2 \le i \le n$

so $\dim L^1 = n$

$\dim L^2 = n - 2$ (~~***~~ dimension $n-1$ is missing!)

$\vdots$

$\dim L^n = 0$ (so L is nilpotent of index n)

<u>Note</u> In reference [11] definition 3 defines <u>null-filiform</u> Leibniz algebra

as $\dim L^i = (n+1) - i$, $1 \le i \le n+1$

So $\dim L^1 = n$

$\dim L^2 = n - 1$ (dimension $n-1$ not missing!)

$\vdots$

$\dim L^n = 1$

$\dim L^{n+1} = 0$ (so L is nilpotent of ~~***~~ index $n+1$)

After Def 2.2 in [55] we have the notion of grL where L is a nilpotent Leibniz algebra as follows: Let L be a nilpotent Leibniz algebra of index s and dimension n,

$$L = L^1 \supsetneq L^2 \supsetneq L^3 \supsetneq \cdots \supsetneq L^{s-1} \supsetneq L^s = (0)$$

Define grL to be the Leibniz algebra

$$grL = L^1/L^2 \oplus L^2/L^3 \oplus \cdots \oplus \underbrace{L^{s-1}/L^s}_{\text{this } = L^{s-1} \text{ since } L^s = 0}$$

Write this as

$$grL = L_1 \oplus L_2 \oplus \cdots \oplus L_{s-1}$$

<u>graded</u> refers to the fact that

$$[L_i, L_j] \subset L_{i+j}$$

where if $i+j \geq s$ then $L_{i+j} = 0$

(we have used the facts that a quotient of a Leibniz algebra by an ideal is also a Leibniz algebra; and direct sums of Leibniz algebras are Leibniz algebras)