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Classification of solvable Lie algebras. **(English summary)**
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Several classifications of solvable Lie algebras of small dimension are known. In this paper one new method of classification of small-dimensional Lie algebras is described. Using this method classifications of three- and four-dimensional solvable Lie algebras (over any field!) are obtained. The three-dimensional case is rather easy, but the four-dimensional case needs some computer calculations. The calculations here use the technique of Gröbner bases. So, unfortunately, it is rather difficult to verify by hand this author's classification of four-dimensional solvable Lie algebras. But the author organizes his calculations very carefully, and I suppose that his classification has no mistakes.

Gröbner bases are used for deriving necessary conditions for isomorphisms of Lie algebras and for finding concrete isomorphisms (by solving some systems of polynomial equations).

In dimension 4 this classification differs slightly from the one found in [J. Patera and H. J. Zassenhaus, Linear Algebra Appl. **142** (1990), 1–17; MR1077969] (where four-dimensional solvable Lie algebras over any perfect field are classified). In this paper some additional Lie algebras are found.

The author also counts the number of solvable Lie algebras of dimension 4 over the finite field $\mathbb{F}_q$, where $q=p^m$ for a prime p.

For dimension 5 the author remarks that his method in this dimension can be rather time consuming and, in some cases (for Lie algebra isomorphism tests), the calculations do not terminate in a reasonable amount of time. *Vladimir V. Gorbatsevich*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.