1. Let \( \{z_n\} \) be a sequence of complex numbers satisfying \( \text{Re } z_n \geq 0 \). Suppose that \( \sum z_n \) and \( \sum z_n^2 \) both converge. Show that \( \sum |z_n|^2 \) converges.

2. Suppose that \( f \) is entire and that \( f(z) \) is real if and only if \( z \) is real. Show that \( f \) can have at most one zero. (Put a circle \( C \) around a couple of supposed zeros of \( f \) and apply the argument principle. Then break up the image curve \( f \circ C \) into three pieces with index 0,0,1 respectively with respect to 0.)

3. Find the number of zeros of (two of these is enough)
   \[
   f_1(z) = 3e^z - z \text{ in } |z| \leq 1 \\
   f_2(z) = e^z/3 - z \text{ in } |z| \leq 1 \\
   f_3(z) = z^4 - 5z + 1 \text{ in } 1 \leq |z| \leq 2 \\
   f_4(z) = z^6 - 5z^4 + 3z^2 - 1 \text{ in } |z| \leq 1
   \]

4. Find a conformal map between the domains \( S \) and \( T \) if (three of these is enough)
   \[
   S = \{ z = x + iy : -2 < x < 1 \} \text{ and } T = \{ |z| < 1 \} \\
   S = T = \text{open upper half plane} \\
   S = \{ z = re^{i\theta} : r > 0, 0 < \theta < \pi/4 \} \text{ and } T = \{ x + iy : 0 < y < 1 \} \\
   S = \{ |z| < 1 \} - [0,1) \text{ and } T = \{ |z| < 1 \} \\
   S = \text{the region between } |z| = 1 \text{ and } |z - 1/2| = 1/2 \text{ and } T \text{ is a half plane} \\
   S = \text{the inside of the right-hand branch of the hyperbola } x^2 - y^2 = 1 \text{ and } T \\
   \text{is the open unit disc. (Map the focus to 0 and the vertex to } -1)\]

5. Find the Laurent expansion for
   \[
   1/(z^4 + z^2) \text{ about } z = 0 \\
   \exp(1/z^2)/(z - 1) \text{ about } z = 0 \\
   1/(z^2 - 4) \text{ about } z = 2
   \]

6. Let \( f_n \) be analytic on an open set \( D \) and suppose \( f_n \) converges uniformly on compact subsets of \( D \). Let \( S = \{ z \in D : f_n(z) = 0 \text{ for some } n \geq 1 \} \) and suppose that \( f \) is not identically zero. Show that \( f \) vanishes exactly at the limit points of \( S \).

7. Prove that if \( u \) is continuous and bounded on the closed upper half plane, harmonic on the open upper half plane, and vanishes on the real axis, then it is a constant.

8. For \( z \) in the upper half plane, let \( u(z) \) be the angle under which the interval \( [0,1] \) is seen from the point \( z \). Show that \( u \) is a harmonic function by finding an analytic function \( f \) such that \( u = \text{Re } f \). (Consider first the angle under which the real axis from 0 to \( \infty \) is seen from \( z \).)