SYMMETRY
A Journey into the Patterns of Nature

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Chapter 1. August: Endings and Beginnings

I think you should find out what mathematics is really about. Read Martin Gardner's column in *Scienfific American*

These numbers (Fibonacci) explain how flowers and shells grow. They are fundamental to the way nature grows things

A symmetry was something active, not passive. The group of symmetries of the starfish interact with each other differently from the symmetries of the triangle

The physical world of symmetries could be translated into an abstract algebraic language. *The Language of Mathematics* by Frank Land.

Unlike German or Russian, this language of mathematics seemed to be a perfect idealized language in which everything made sense and there were no irregular verbs or nonsensical exceptions

Nature's Language

The coral surface is covered with one of nature's most remarkable symmetrical animals. Why is symmetry so pervasive in nature?

It provides a way for animals and plants to convey a multitude of messages, from genetic superiority to nutritional information. For an insect such as a bee, symmetry is fundamental to survival

For the bee, survival of the fittest means becoming an expert at symmetry. Symmetry is the language used by the flower and bee to communicate with each other

The asymmetry of our internal organs is still something of a mystery and only goes to reinforce the wonder at how symmetrical the exterior is

Animals have been drawn to mirror symmetry because of the superior motor skills it offers. The food goes to the animal with the most symmetry because it's going to get to the dinner table first (instead of becoming dinner)

The hexagonal lattice that the bees use to store their honey exploits another facet of symmetry, allowing the colony to pack the most honey into the greatest space without wasting too much space in building the walls

When a soap bubble forms it tries to assume the shape of a perfect sphere (the three dimensional object with the most symmetry). The sphere is the shape with the smallest surface area that can contain a given volume of air, hence it uses the least energy

The true picture of a drop of water falling from the sky is a sphere. Molten lead is dropped from a great height into buckets of cold water to make perfectly symmetic balls

A diamond gets its strength from its highly symmetrical arrangement of carbon atoms.

The word ‘symmetry’ conjures to mind objects which are well balanced, with perfect proportions. Such objects capture a sense of beauty and form

The human mind is constantly drawn to anything that embodies some aspect of symmetry. Our brain seems programmed to notice and search for order and structure

Symmetry is about connections between different parts of the same object. It sets up a natural internal dialogue in the shape

The Symmetry Seekers

For the mathematician, the pattern searcher, understanding symmetry is one of the principal themes in the quest to chart the mathematical world

Symmetry is a slippery concept. What exactly is it? When do two objects have the same symmetries and when are they different?

It took a stunning breakthrough during the revolutionary fervor of 19th century Paris for a new language to emerge that could capture the true meaning of the word. It was called **group theory**

This new language became the seed for a mathematical revolution which would match, in its implications the political upheaval taking place in the streets of Paris.

Suddenly, mathematics had the tools to build ships to set sail for the very limits of the world of symmetry

Just like the fact that natural numbers can be written as a product of primes, every symmetrical object can be divided into certain smaller objects whose collection of symmetries was indivisible.

When mathematicians finally fully grasped the idea of what made a symmetrical object indivisible, they saw the prospect of producing a 'periodic table' consisting of all the different possible indivisible symmetrical objects

Such a table would list all the building blocks out of which all possible symmetrical objects can be constructed

The rotational symmetries of a prime-sided polygon were the first objects to be listed in the table.

The mathematicians of the 19th century discovered that the **icosahedron** was a (stranger) object whose symmetrys could not be reduced to smaller objects

The mathematicians of the 19th and 20th centuries unearthed and added more and more indivisible symmetrical objects to this mathematical periodic table. The list just kept on growing

In the 1970s, two distinct teams of mathematicians worked on this project. One team specialized in finding more and more exotic mathematical objects whose symmetries were indivisible

The second team worked at the other end, exploiting the limits of symmetry. Their efforts were focused on showing what was not possible

Would the second team ever be able to show the first team that there was no longer anywhere new to explore? Or might there always be uncharted waters?

Toward the end of the 1970s, a complete taxonomy of symmetry was in sight—a periodic table containing all the building blocks of symmetry was emerging

Most mathematicians were thrilled at the prospect of a proof that the symmetry seekers had found all the building blocks. But not all were happy

The world of symmetry had been circumnavigated. However, there were no press conferences to announce that the proof had been completed

No one is even too clear who had actually finished it. Some still question whether it truly has been completed

For the mathematical community, however, it was big news.

For the first time in mathematics, here was a proof which involved such a collective effort that it was impossible and meaningless to put a single name to it

Unlike any other proof, this one was so immense that it was unclear whether any one person could claim to have read all the ten thousand pages over 500 different journals that completed its account

The proof, though vast and complex, is full of jewels that would have appealed to Hardy's sense of aesthetics. ("A mathematical proof should resemble a simple and clear-cut constellation, not a scattered cluster in the Milky Way")

With crystal-clear logic, the new mathematical proof explained why all those symmetries should be out there and why we weren't going to find any more

Members of the first time produced the 'Atlas': mathematical charts the topography of each new group of symmetries encountered. This Atlas of symmetry, published in 1985, became a Rosetta Stone for many scientists

Now, two millennia after the Ancient Greeks had started to explore shapes with symmetry, mathematics had got its own periodic table