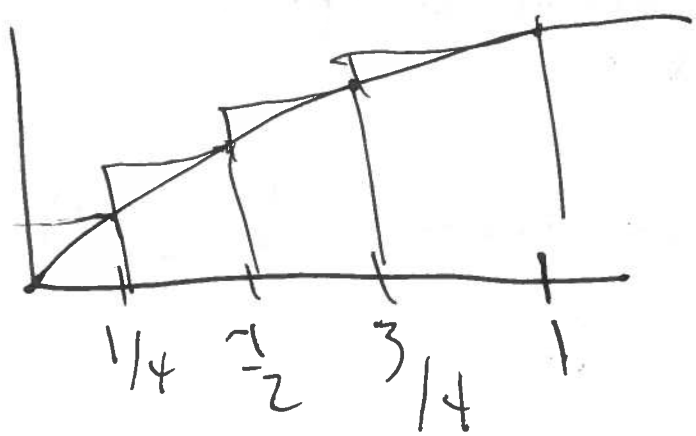
Homework 11

5.1) 4, 10, 4 5.3) 2, 14, 44, 54

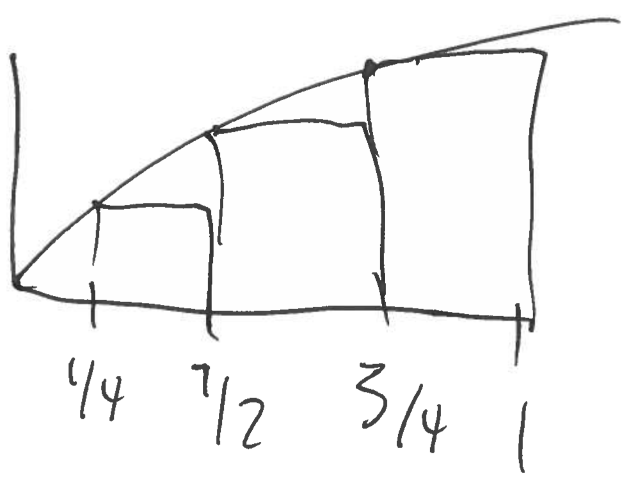
5.2) 18, 22, 32, 36 5.4) 4, 14, 18, 28

5.1.4)



$$R_4 = \frac{1}{4}\left(\sqrt{1/4} + \sqrt{1/2} + \sqrt{3/4} + \sqrt{1}\right)$$

$\approx .7683$ over



$$L_4 = \frac{1}{4}\left(\sqrt{0} + \sqrt{1/2} + \sqrt{3/4}\right)$$

$\approx .5163$ under.

See code on next page
with output.

True answer: $\log(256) - 3 \approx 2.5452$

```
nvals = [10 30 50 100 1000]; % make array of possible n values:

for j = 1:length(nvals) %iterate over n values
    n = nvals(j);
    xvals = linspace(1,4, n+1); % returns an array of the interval 1 to 4
                                 %divided into n+1 interval, i.e. our endpoints

    % xvals looks like [1 1.333 ... 4] so for left endpoints we want to
    % toss the last point, right endpoints we want to toss the first
    % in MATLAB, this is:

    xvals_right = xvals(2:end); % chops off first element
    xvals_left = xvals(1:end-1); % chops off last element

    deltax = (4-1)/n; % compute width of each rectangle

    f = @(x) log(x); % define our function f using MATLAB syntax

    f_right = f(xvals_right); % compute our right endpoint rectangle heights
    f_left = f(xvals_left); % same for left

    % notice we can put an array into a function, one of MATLAB's strenghts

    Rn = deltax*sum(f_right) % Rn = sum deltax * f(xj)
    Ln = deltax*sum(f_left) % same for Ln
end
```

```
>> riemann
n =
    10
Rn =
    2.7475
Ln =
    2.3316
n =
    30
Rn =
    2.6139
Ln =
    2.4752
n =
    50
Rn =
    2.5865
Ln =
    2.5034
n =
   100
Rn =
    2.5659
Ln =
    2.5243
n =
        1000
Rn =
    2.5473
Ln =
    2.5431
```

approaches value we desire

5.1.18) $f(x) = x^2 + \sqrt{1+2x}$ $\quad 4 \le x \le 7$

Wrong problem

sorry.

$$\Delta x = \frac{7-4}{n} = \frac{3}{n}.$$

$$x_j = x_0 + j\Delta x = 4 + \frac{j3}{n}.$$

$$\int_4^7 f(x) = \lim_{n\to\infty} \sum_{j=1}^{n} f(x_j^*) \Delta x.$$

$$\boxed{\text{Choose } x_j^* = x_j.}$$

$$= \lim_{n\to\infty} \sum_{j=1}^{n} f(x_j) \Delta x$$

$$= \lim_{n\to\infty} \sum_{j=1}^{n} \left(4 + \frac{3j}{n}\right)^2 + \sqrt{1 + 2\left(4 + \frac{3j}{n}\right)} \cdot \left(\frac{3}{n}\right)$$

5.2.18)

$$\lim_{n\to\infty}\sum_{j=1}^{n}\frac{\cos x_i}{x_i}\Delta x = \lim\sum_{i=1}^{n} f(x_i)\Delta x.$$

for $f(x) = \frac{\cos(x)}{x}.$

Thus, this is equal to $\displaystyle\int_{\pi}^{2\pi}\frac{\cos(x)}{x}\,dx.$

5.2.22) This problem is long and hard.

$$\Delta x = \frac{4-1}{n} = \frac{3}{n}.$$

$$x_i = x_0 + i\Delta x = 1 + \frac{3i}{n}.$$

$$f(x_i) = x_i^2 + 2x_i - 5$$

$$= \left(1 + i\frac{3}{n}\right)^2 + 2\left(1 + i\frac{3}{n}\right) - 5$$

$$= i^2\frac{9}{n^2} + i\frac{12}{n} - 2 \quad \text{after algebra}$$

Thus:

$$\int_1^4 x^2 + 2x - 5 \, dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} \left(i^2 \cdot \frac{9}{n^2} + i \cdot \frac{12}{n} - 2\right)\frac{3}{n}$$

$$= \lim \left[\sum_{i=1}^{n} \frac{27}{n^3} \cdot i^2 + \frac{36}{n^2} \cdot i - \frac{6}{n}\right]$$

$$= \lim \left[\frac{27}{n^3}\sum i^2 + \frac{36}{n^2}\sum i - \frac{6}{n} \cdot \sum 1\right]$$

$$= \lim \left[\frac{27}{n^3} \frac{n(n+1)(2n+1)}{6} + \frac{36}{n^2} \cdot \frac{n(n+1)}{2} - \frac{6}{n} \cdot n\right]$$

$$= \frac{54}{6} + \frac{36}{2} - 6$$

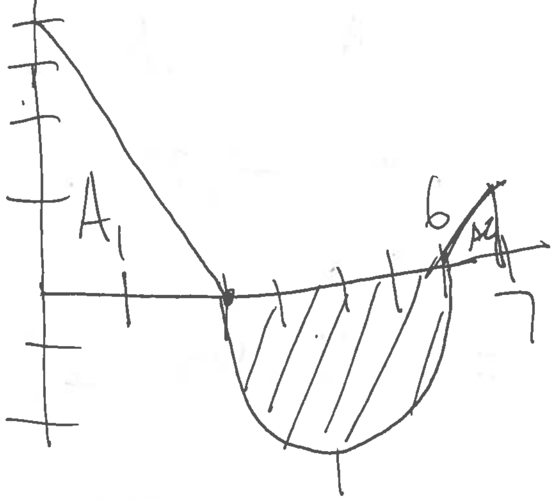
$$= 21.$$

5.2.32

$\int_2^6 g(x)\,dx$.

Clearly we just have a semicircle.

b.)



Thus,

$\int_2^6 g(x) = -$ Area of semicircle

$= -\pi r^2$

$= -\pi(2)^2$

$= -4\pi$.

~~5.2.36~~

a.) ~~the~~ ~~[illegible]~~

$\int_0^2 g(x)\,dx$ corresponds to the area under the first triangle.

$\int_0^2 g(x)\,dx = A_1 = \frac{1}{2}\cdot b\cdot h$

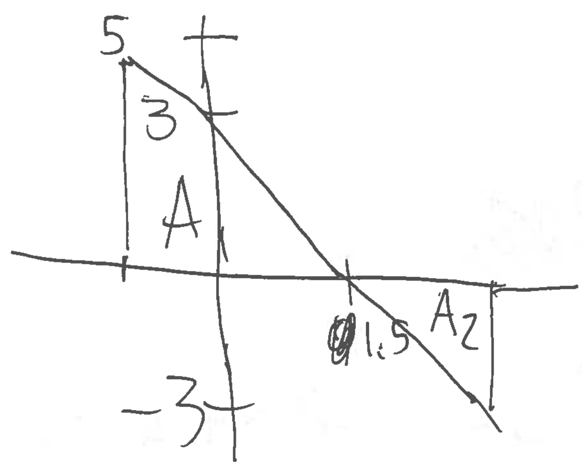
$= \frac{1}{2}\cdot 2\cdot 4 = 4$.

c. $\int_0^7 g(x) = A_1 + A_{semicirc} + A_2$

$$\underset{4}{A_1} \quad \underset{-4\pi}{A_{semicirc}} \quad \underset{\frac{1}{2}\cdot b\cdot h = \frac{1}{2}\cdot 1\cdot 1 = \frac{1}{2}}{A_2}$$

$= 4 - 4\pi + \frac{1}{2}$

5.2.36) $\int_{-1}^{3} (3-2x)\,dx =$



$A_1 = \frac{1}{2}\cdot b\cdot h = \frac{1}{2}\cdot 5\cdot (2.5)$
$= 6.25$

$A_2 = \frac{1}{2}\cdot b\cdot h = \frac{1}{2}(-3)(1.5) = -2.25$

$A_1 + A_2 = 4$

5.3.2.1

$$\int_1^2 x^{-2} = \frac{x^{-1}}{(-1)}\Bigg|_{x=1}^{x=2}$$ by the power rule for integrals

$$= -\frac{1}{2} - \frac{-1}{1} = \frac{1}{2}.$$

5.3.14

$$\int_0^{\pi} \sec\theta \tan\theta \, d\theta$$

Remember:

$$\frac{d}{d\theta}(\sec\theta) = \sec\theta \tan\theta$$

Therefore:

$$\int_0^{\pi} \sec\theta \tan\theta \, d\theta = \sec\theta \Bigg|_{\theta=0}^{\theta=\pi}$$

$$= \sec\pi - \sec 0$$

$$= -1 - 1 = -2.$$

5.3.44.) $\int v(v^2+2)^2\,dv$.

$$= \int v(v^4+4v^2+4)$$

$$= \int (v^5+4v^3+4v)\,dv$$

$$= \frac{v^6}{6} + \frac{4v^4}{4} + \frac{4v^2}{2} + C.$$

5.3.54) $100 + \int_0^{15} n'(t)$

Recall $\int n'(t)$ describes net change in n.

Thus we have the total change in bees from 0 to 15 weeks.

Thus, $100 + \int_0^{15} n'(t) =$ # bees after week 15.

5.4.4) $g(x) = \int_0^x f(t)\,dt$.

a.) $g(0) = \int_0^0 = 0$.

$g(6) = \int_0^6 f(t)\,dt$.

$= 0$ because the function is exactly symmetric. That is, areas cancel.

b.) $g(1) \approx 3$

$g(2) \approx 3 + 2 = 5$

$g(3) \approx 3 + 2 + 1 = 6$

$g(4) \approx 3 + 2 + 1 - 1 = 5$

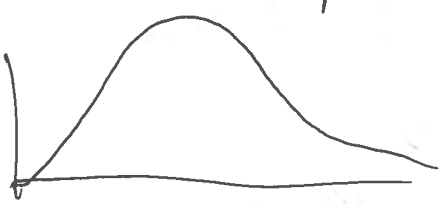
$g(5) \approx 3 + 2 + 1 - 1 - 2 \approx 3$

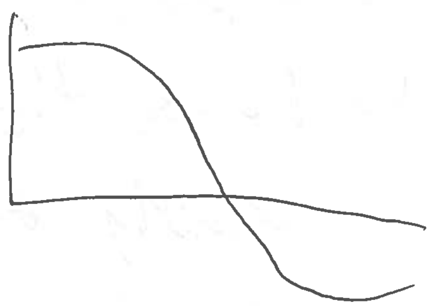
$g(6) = 0$.

c.) $g'(x) = f(x)$. Thus, increasing when $f > 0$

i.e. x between 0 and 3,

d.) critical pt $= x = 3$

e.) 

f.) 

matches.

5.4.14) $h(x) = \int_0^{x^3} \sqrt{1+r^3}\, dr$.

Let $g(x) = \int_0^{x^3} \sqrt{1+r^3}\, dr$,

Then we have:

$h(x) = g(u(x))$ where $u = x^3$.

Then: $h'(x) = g'(u) \cdot u'(x)$

$g'(u) = \sqrt{1+u^3}$ by FTC.

and $u' = 3x^2$ ~~by~~. Thus: $h'(x) = \underline{\sqrt{1+(x^3)^3} \cdot 3x^2}$

5.4.16) ~~ant~~ $f(u) = \int_{\sin x}^{\cos x} (1+v^2)^{10}\, dv.$

$$= \int_0^{\cos x} (1+v^2)^{10}\, dx + \int_{\sin x}^{0} \cdot\, dx$$

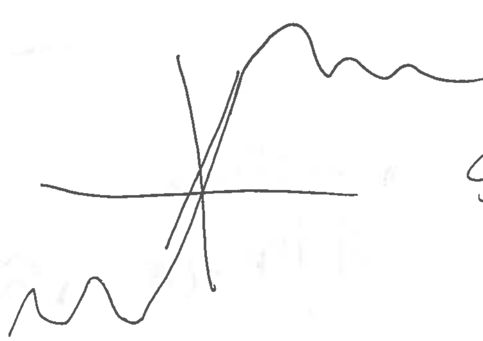
$$= \int_0^{\cos x} (1+v^2)^{10}\, dx - \int_0^{\sin x} (1+v^2)^{10}\, dx.$$

by FTC,

$$f'(x) = (1+\cos^2 x)^{10} \cdot \frac{d}{dx}(\cos x) - (1+\sin^2)^{10} \cdot \frac{d}{dx}(\sin x)$$

$$= -(1+\cos^2 x)^{10} \cdot \sin(x) - (1+\sin^2 x)^{10} \cdot \cos(x)$$

5.4.28.a)



$Si(x)$

b.) $Si'(x) = 0$ @ $\frac{\sin x}{x} = 0$.

Occurs @ $x = 0, \pi, 2\pi$.

but

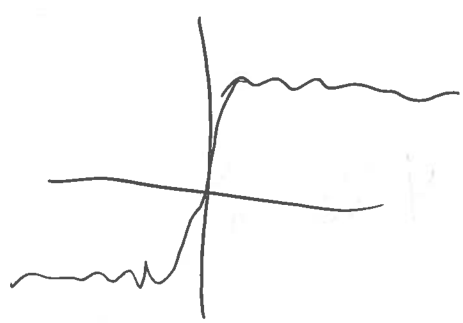
$$Si''(x) = \frac{x\cos(x) - \sin(x)}{x^2}.$$

only $\frown$ concave down at

$0, 2\pi, -2\pi$ etc.

c.) $x \cdot \cos(x) - \sin(x) = 0$. $x \approx 4.493$

d.) Yes. $\frac{\pi}{2}, -\frac{\pi}{2}$.



e.) $Si(x) = 1$ on Wolfram Alpha gives: $x \approx 1.06$.