HW3 Solutions

2.5 4, 8, 16, 34, 40

2.4 4, 14, 16, 22, 28

2.5.4

a.) Although it isn't 100% clear from the graph, we can assume the graph stays at $y=2$ as $x \to \infty$.

Thus, $\lim_{x \to \infty} g(x) = 2$.

b.) Similally, $\lim_{x \to -\infty} g(x) = -2$ as the graph seems to flatten out as we continue left (and beyond) of the graph.

c.) As $x \to 3$, the graph blows up, but matches on both sides, so

$$\lim_{x \to 3} g(x) = +\infty.$$

d.) Similarly, as $x \to 0$, the graph goes to $-\infty$ from both sides, meaning we can conclude

$$\lim_{x \to 0} g(x) = -\infty.$$

e.) From the right side of -2, the graph seems to become very negative. Therefore

$$\lim_{x \to -2^+} = -\infty.$$

f.) There are only 4 asymptotes here: two vertical and two horizontal.

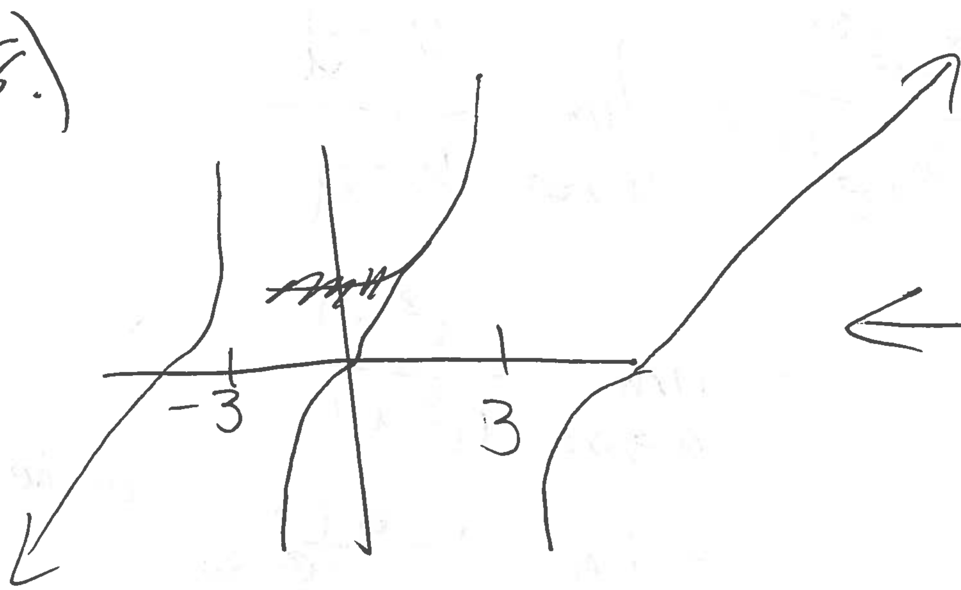
Vertical $x = 3$
$x = -2$.

Notice -2 may ~~b~~ confuse some people but we defined an asymptote to be if the limit on either side

Since (a) (b) are true,

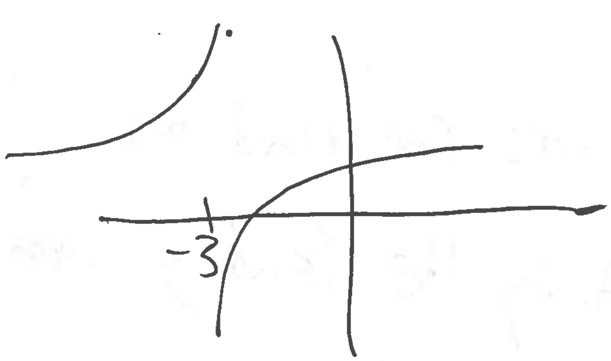
$y = 2$ must be horizontal
$y = -2$
asymetotes.

2.5 8.)



One example $\leftarrow$

2.5. 16) $\lim_{x \to -3^-} \frac{x+2}{x+3}$. This clearly describes a vertical asymetote at $x = -3$. Notice, to the left, it is pos. to the right, it is neg.



Thus, $\lim_{x \to -3^-} \frac{x+2}{x+3} = +\infty$.

2.5.34) $$\lim_{x\to\infty} \frac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}}.$$

Let $u = e^{3x}$.

We now have,

$$\lim_{x\to\infty} \frac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}} = \lim_{u\to\infty} \frac{u-\frac{1}{u}}{u+\frac{1}{u}}$$

$$= \lim_{u\to\infty} \frac{u^2-1}{u^2+1}.$$

divide by u^2

$$= \lim_{u\to\infty} \frac{1-\frac{1}{u^2}}{1+\frac{1}{u^2}} \quad \frac{1}{u^2}\to 0$$

$$= 1.$$

Note, when we had

$\lim_{u\to\infty} \frac{u^2-1}{u^2+1}$ we can read off the limit by noticing the leading terms have the same power and coefficient $\frac{1}{1}=1$.

2.5.40) $y = \frac{x^2+1}{2x^2-3x-2}$,

First, lets consider horizontal asymptotes by examining the behavior of $f(x)$ as $x \to \infty$ and $x \to -\infty$.

Note, the leading terms have the same power of x meaning

$$\lim_{x \to \infty} f(x) = \text{a number}.$$

Since they grow at roughly the same rate. Specifically:

$$\lim_{x \to \infty} f(x) \approx \frac{(\text{big } \#)^2 + \text{small stuff}}{2(\text{big } \#)^2 + \text{small stuff}}$$

$$\approx \frac{1}{2}.$$

Note a negative number squared

is also positive so we can also conclude.

$$\lim_{x\to\infty} f(x) = \frac{1}{2}.$$

This gives us a horizontal asymptote at $y=\frac{1}{2}$.

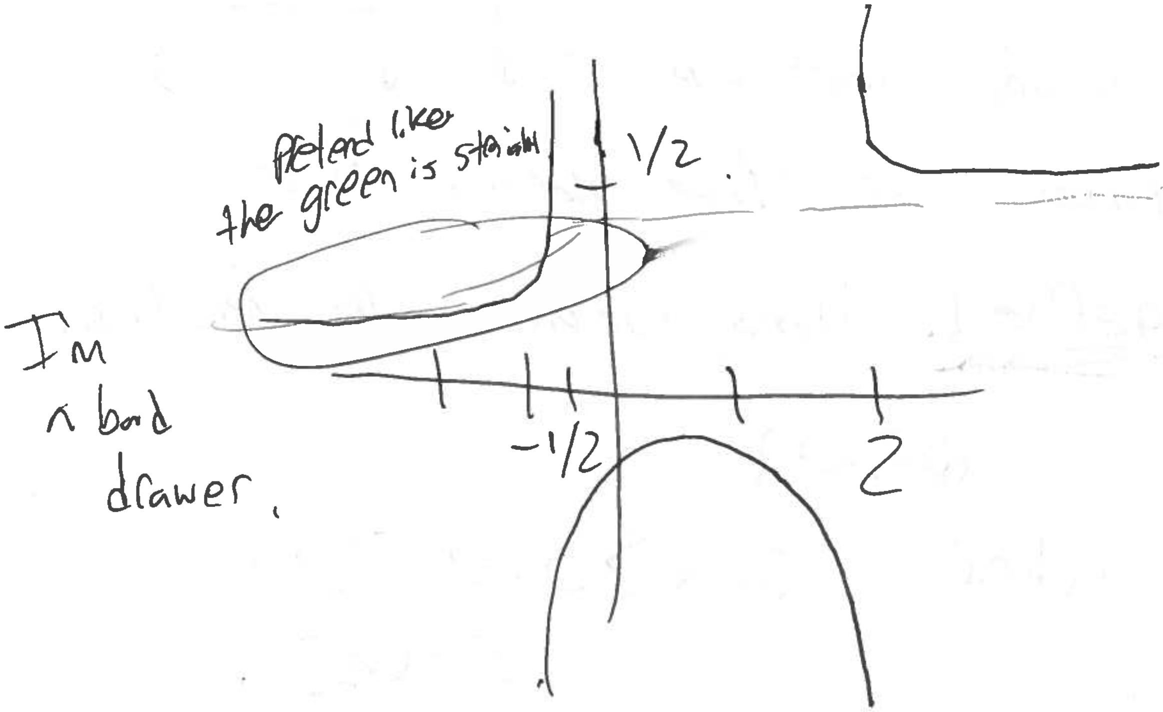
What about vertical? Occurs when denominator $=0$.

Using the quadratic equation or factoring we find

$$f(x) = \frac{x^2+1}{2x-3x-2} = \frac{x^2+1 \text{ (not factorable)}}{(x-2)(x+\frac{1}{2})}.$$

If one of these looked like it cancelled, we would have a removable discontinuity, so clearly $x=2$ and $x=-\frac{1}{2}$ are instead vertical asymptotes.

The plot of this graph is roughly:



2.4.4) The graph is continuous on any interval that the limit equals the function evaluation. We can include endpoints if it is left or right continuous.

Thus: $[-4,-2) \cup (-2,2) \cup [2,4) \cup (4,6) \cup (6,8)$.

2.4.14.) $2\sqrt{3-x}$ Continuous on $(-\infty, 3]$?

Consider that we said $\sqrt{\ }$ is continuous on the domain it is defined. Here, when is the function defined?

when: $3-x \geq 0 \Rightarrow 3 \geq x$

$\Rightarrow x \leq 3$.

$= x \in (-\infty, 3]$.

2.4.16). Is

$$\begin{cases} \frac{x^2-x}{x^2-1} & x \neq 1 \\ 1 & x = 1 \end{cases}$$

continuous?

Obviously the only point we need check is $x=1$.

Particularly, for $f(x)$ to be continuous at $x=1$, we need:

$$\lim_{x \to 1} f(x) = f(1) = 1.$$

What is this limit?

$$\lim_{x\to 1} \frac{x^2-x}{x^2-1} = \lim \frac{x\cancel{(x-1)}}{\cancel{(x-1)}(x+1)}$$

$$= \lim_{x\to 1} \frac{x}{x+1} = \frac{1}{2}.$$

Clealy $\lim_{x\to 1} f(x) \neq f(x)$ so this function is <u>discontinuous</u> at $x=1$.

2.4.22) $h(x) = \dfrac{\sin(x)}{x+1}$

We know $\sin(x)$ is continuous everywhere.

Dividing by 0 is a problem.

But our theorem says if have $h = \dfrac{f(x)}{g(x)}$ and $g(x) \neq 0$, then if f, g are each continuous, h is too.

Is x+1 continuous? Yes,

How do we deal with dividing by 0?

Is h(-1) even defined? No!

Therefore is x = -1 even in the domain of h(x)? No!

Therefore, h(x) is continuous for all x in its domain, where the domain is:

$$D = \{x \in \mathbb{R} : x \neq -1\}.$$

2.4.28) $\lim_{x \to \pi} \sin(x + \sin x)$.

We have a Theorem that says basically:

if f(x) is continuous and

g(x) is continuous

then f(g(x)) is continuous.

So here, consider

$$f(x) = \sin(x)$$
$$g(x) = x + \sin(x).$$

Clearly, $f(x), g(x)$ are continuous since there are no jumps / gaps / asymptotes.

Thus, $h(x) = f(g(x)) = \sin(x + \sin(x))$ is continuous, meaning

$$\lim_{x \to \pi} h(x) = h(\pi)$$
$$= \sin(\pi + \sin(\pi))$$
$$= \sin(\pi + 0)$$
$$= \sin(\pi)$$
$$= 0.$$