HW 6 Solutions

3.3: 6, 20, 36
3.4: 18, 24, 38, 68, 82.

3.3.6] $e^{\theta}(\tan\theta - \theta) = g(\theta)$

$\frac{d}{d\theta}(g(\theta)) =$ Product rule

$\frac{d}{d\theta}(e^{\theta}) = e^{\theta}$

$\frac{d}{d\theta}(\tan\theta - \theta) = \sec^2\theta - 1$

$\frac{d}{d\theta}(g(\theta)) = e^{\theta}(\tan\theta - \theta) + e^{\theta}(\sec^2 - 1)$

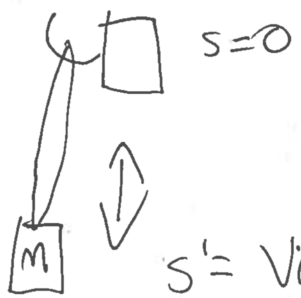
3.3.20] $y = e^x\cos x$

$y' = e^x \cdot \cos(x) + e^x \cdot (-\sin(x))$

$y'(0) = e^0\cos(0) - e^0\sin(0)$
$= 1.$

$\therefore$ $y - 1 = 1(x - 0) \Rightarrow y = x + 1.$

3.3, 26



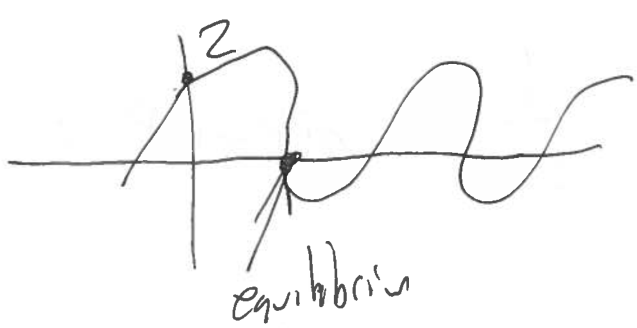
$$s = 2\cos t + 3\sin t, \quad t \geq 0$$

$$s' = \text{velocity} = -2\sin(t) + 3\cos t$$

$$s'' = \text{accel} = -2\cos(t) - 3\sin(t)$$

equilibrium =>

S(t)



$$S(t) = 0$$

$$2\cos(t) + 3\sin(t) = 0.$$

$$\frac{1}{\cos(t)}\left(2\cos(t) + 3\sin(t)\right) = 0.$$

$$2 + 3\tan(t) = 0$$

$$\tan(t) = -\frac{2}{3}$$

$$\tan(t) = t\tan^{-1}\left(\frac{-2}{3}\right) \ldots$$

~~Max displacement:~~

~~Max velocity: $-2\sin(t) + 3\cos(t)$~~

~~Max = 2+3 = 5~~

* This problem is actually hard to do by hand. Graphing is ok.

Max displacement: $\sqrt{2^2+3^2} = \sqrt{13}$ cm.

Max ~~velocity~~ speed: $\sqrt{2^2+3^2} = \sqrt{13}$ cm/s.

3.4.18] $y = e^{-2t}\cos 4t$

$$\frac{dy}{dt} = \frac{d}{dt}(e^{-2t}) \cdot \cos 4t + \frac{d}{dt}(\cos 4t) \cdot e^{-2t}$$

$$= -2e^{-2t} \cdot \cos 4t + 4\sin(4t) \cdot e^{-2t}.$$

3.4.24]

$$G(y) = \left(\frac{y^2}{y+1}\right)^5$$

$f(y)$

$$G' = 5(f(y))^4 \cdot f'(y),$$

$$f'(y) = \text{quotient rule: } \frac{2y \cdot (y+1) - (1)(y^2)}{(y+1)^2}.$$

$\therefore$

$$G'(y) = \underline{5\left(\frac{y^2}{y+1}\right)^4} \cdot \frac{2y(y+1) - y^2}{(y+1)^2}.$$

3.4.38] $y = \cos^2 x$

$= (\cos x)^2.$

$\therefore y' = 2\cos x \cdot (\cos x)'$

$= 2\cos(x) \cdot -\sin x,$

$$y'' = \frac{d}{dx} = -2\cos(x)\sin(x) = \text{Product rule}$$

$$= +2\sin(x)\cdot\sin(x) + (-2)\cos(x)\cdot\cos x$$

$$= 2\sin^2(x) - 2\cos^2(x)$$

$$= 2(\sin^2(x) - \cos^2(x)),$$

3, 4, 68

$$f(x) = xe^{-x}$$

I messed this up a little in class.

by product rule

$$f'(x) = e^{-x} + -xe^{-x}.$$

$$f''(x) = -e^{-x} - (e^{-x} - xe^{-x}).$$

$$= -2e^{-x} + xe^{-x}$$

$$f'''(x) = -2e^{-x}(-1) + e^{-x} - xe^{-x}.$$

$$= 3e^{x} - xe^{-x}.$$

Note, $(-1)^{n+1}$ gives us negative for even n and positive for odd n. $(-1)^n$ does the opposite

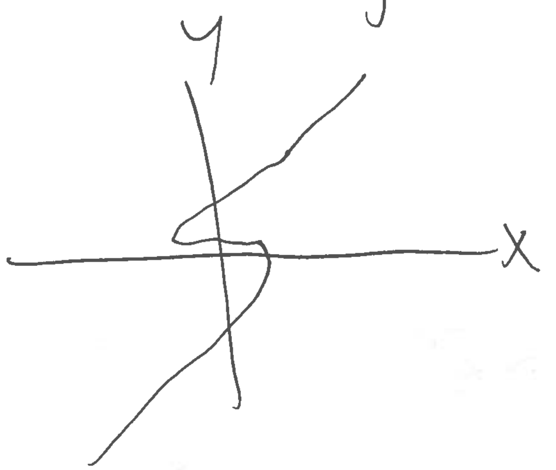
$$\boxed{f^{(n)}(x) = (-1)^{n+1} n\cdot e^{-x} + (-1)^n xe^{-x},}$$

$\therefore \quad f^{(1000)}(x) = (-1)^{1001} 1000 e^{-x} + (-1)^{1000} x e^{-x}$

$$\boxed{= -1000 e^{-x} + x e^{-x}.}$$

3.4.82] $x = 2t^3 + 3t^2 - 12t$

$y = 2t^3 + 3t^2 + 1$



$$\boxed{\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}}$$

Horiz: if $\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dT} = 0$

$\frac{dy}{dt} = 6t^2 + 6t = 6(t)(t+t)$

$\Rightarrow \boxed{\begin{array}{l} t = -1 \\ t = 0 \end{array}}$

Veltical:

$$\frac{dy}{dx} = \infty \Rightarrow \frac{dx}{dt} = 0$$

$$\frac{dx}{dt} = 6t^2 + 6t - 12$$

$$= 6(t^2 + t - 2)$$

$$= 6(t+2)(t-1)$$

$$\Rightarrow t = -2$$

$$t = 1.$$