Homework 9 Solutions

4.2] 26, 30, 38, 52
4.3] 8, 22, 30

4.2.6] Critical # c when

$f'(c)=0$ or $f'(c)$ DNE

$f(x) = x^3+x^2+x$

$f'(x) = 3x^2+2x+1$,

Clearly exists everywhere, where $=0$?

$0 = 3x^2+2x+1$

$= \dfrac{-2 \pm \sqrt{4-4(3)(1)}}{2(3)}$

= complex therefore no critical numbers.

4.2.30]

$h(p) = \frac{p-1}{p^2+4}$, $h'(p) = \frac{-p^2+2p+4}{(p^2+4)^2}$

by ~~[illegible]~~ quotient rule.

$h'(p) = 0$?

$-p^2+2p+4 = 0$

$p = 1 \pm \sqrt{5}$.

$h'(p)$ DNE?

$p^2+4=0$

$p^2=-4$

$p = \pm 2i \leftarrow$ not a critical #.

$\boxed{4.2.38}$ $f(x) = x^{-2}\ln(x)$

$f'(x) = x^{-3} - 2\ln(x) \cdot x^{-3}$

by product rule

$$f'(x) = \frac{1 - 2\ln(x)}{x^3}$$

$f'(x) = 0$?

$1 - 2\ln(x) = 0$

$\ln(x) = \frac{1}{2}$

$x = e^{1/2}$

$f'(x)$ DNE?

$x = 0$

4.2.52) $f(x) = x - \ln x$ on $[\frac{1}{2}, 2]$

Critical #'s?

$f'(x) = 1 - \frac{1}{x} = \frac{x-1}{x}$

$f'(x)$ DNE @ $x = 0$, but not in interval

$f'(x) = 0$ @ $x = 1$. Critical #.

Must check critical #'s and end points

$$f(\tfrac{1}{2}) = \tfrac{1}{2} - \ln(\tfrac{1}{2}) \approx 1.193$$

$$f(1) = 1 - \ln(1) = 1$$

$$f(2) = 2 - \ln(2) \approx 1.307.$$

Clearly $x=1$ abs. min.

$x=2$ abs max.

4.3.8] $f(x) = 4x^3 + 3x^2 - 6x + 1$

$$f'(x) = 12x^2 + 6x - 6.$$

$$= 6(2x^2 + x - 1).$$

$$= 6(2x-1)(x+1)$$

$f' = 0$ @ $x = \frac{1}{2}$, $x = -1$

critical #'s!

$f'(x)$

increasing decreasing

$+$ $-$ $-$

-1 $\frac{1}{2}$

$f''(x) = 24x + 6.$

$= 6(4x+1)$

$f''(0)$ @ $x = -\frac{1}{4}$, inflection point

f''

concave down concave up.

$-$ $+$

$-\frac{1}{4}$

Abs min/max occur at critical #'s.

$x = -1$. By 1st deriv test: max

or By second deriv test concave down: max.

$x = \frac{1}{2}$ by 1st deriv test: $\searrow \nearrow$ min

or by 2nd deriv test: Concave up $\cup$. min.

4.3.22)

$f(x) = 2+3x - x^3$.

$f'(x) = 3 - 3x^2$

$= 3(1 - x^2)$

$x = \pm 1$ ← $f'(0)$ critical numbers.

$f''(x) = -6x$

$f''(0) \to x = 0$. inflection point

f': $-$ | $+$ | $-$
at -1 and 1

f'': $-$ | $+$
at 0

By 1st deriv test:

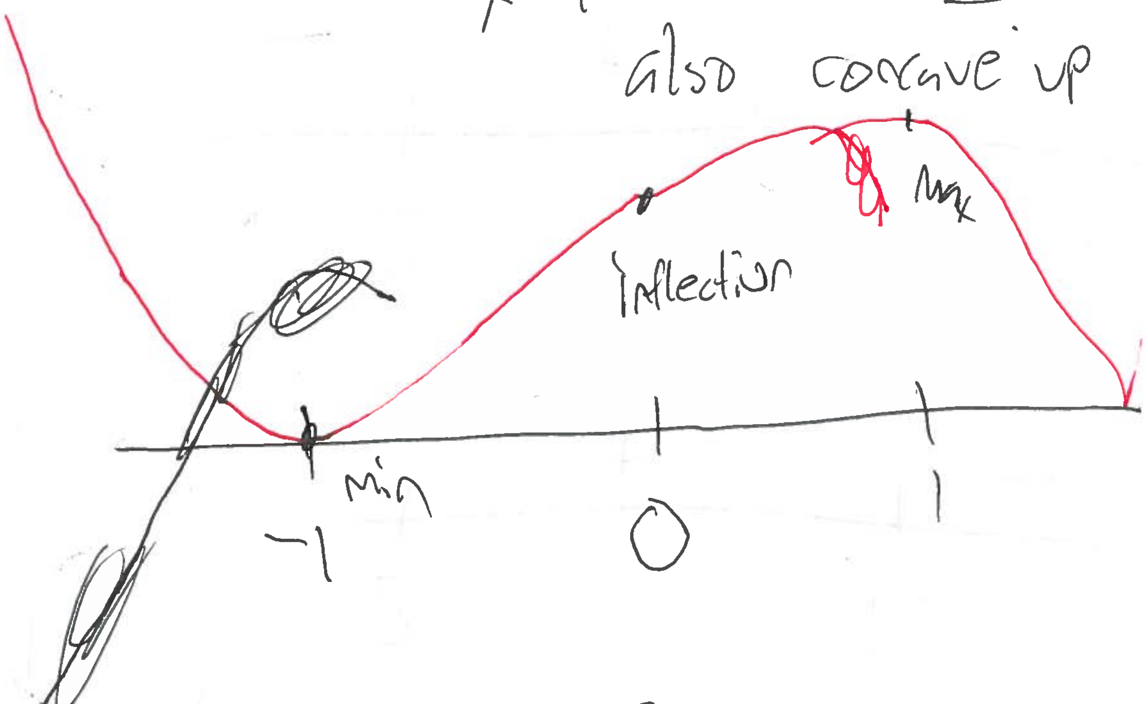
$x=-1$ min

also concave down

By same logic:

$x=1$ max

also concave up



4.3.34] $f(x) = \dfrac{x^2}{(x-2)^2}$.

Horiz asymp: $\lim_{x\to\infty} f = 1$.

Vertical: $x=2$ blows up.

(after simplifying)

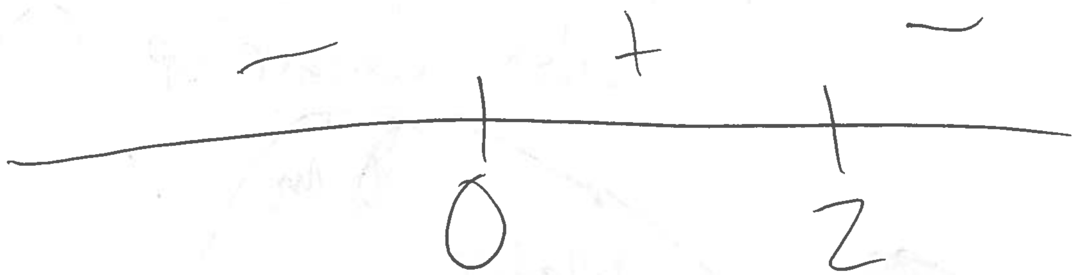
$$f'(x) = \frac{-4x}{(x-2)^3},$$

$f'(x)=0$ @ $x=0$,
$f'(x)$ DNE @ $x=2$.

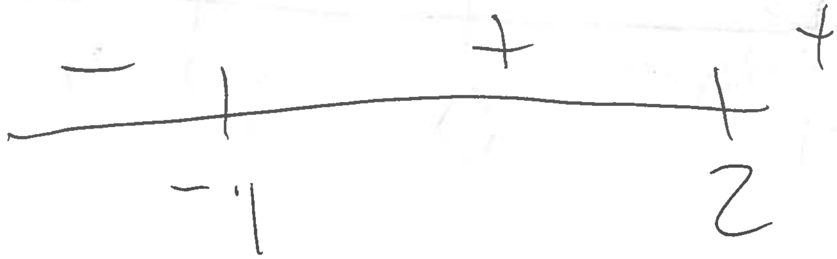
$$f''(x) = \frac{8(1+x)}{(x-2)^4}.$$

$f''(x)=0$ @ $x=-1$
$f'(x)$ DNE @ $x=2$.

f'



f''



$x=0$ \/ <u>Min</u>.

$x=2$ weird asymptote.