

Name: \_\_\_\_\_

Quiz Score: \_\_\_\_/10

Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

- 3 1. Evaluate the following definite integral:

$$\int_0^1 x^{5/6} dx.$$

**Solution:** This is a simple application of the power rule for integrals, which states that if  $f(x) = x^n$ , then  $F(x) = x^{n+1}/(n+1)$  is an antiderivative. Here, we have  $f(x) = x^{5/6}$ , then we have the antiderivative  $F(x) = \frac{x^{11/6}}{11/6}$ , but we have a **definite** integral, meaning we need to evaluate the antiderivative:

$$\int_0^1 x^{5/6} dx = \left. \frac{x^{11/6}}{11/6} \right|_{x=0}^{x=1} = \frac{1^{11/6}}{11/6} - 0 = \frac{6}{11}.$$

- 3 2. Compute the following:

$$\frac{d}{dx} \left\{ \int_x^1 \cos \sqrt{t} dt \right\}.$$

State your reasoning and/or whichever theorems you use. *Hint:* are the bounds of the integration right to apply the theorem?

**Solution:** The Fundamental Theorem of Calculus states:

$$\frac{d}{dx} \left\{ \int_a^x f(t) dt \right\} = f(x).$$

Notice, we do not have this particular form, as our integral is of the form  $\int_x^a$ . Recall we have the relationship:

$$\int_a^x f(t) dt = - \int_x^a f(t) dt,$$

which means we can transform our original problem into:

$$\frac{d}{dx} \left\{ \int_x^1 \cos \sqrt{t} dt \right\} = \frac{d}{dx} \left\{ - \int_1^x \cos \sqrt{t} dt \right\} = - \frac{d}{dx} \left\{ \int_1^x \cos \sqrt{t} dt \right\}.$$

Notice, the derivative is now exactly of the form of FTC, meaning we have:

$$- \frac{d}{dx} \left\{ \int_1^x \cos \sqrt{t} dt \right\} = - \cos \sqrt{x}.$$

- 4 3. Evaluate the following indefinite integral:

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx.$$

**Solution:** Here, we must perform a  $u$  substitution. Many of you tried several different substitutions, but I didn't see any that worked except for the one I had in mind:  $u = \sqrt{x}$ . Notice, this means that:  $du = dx/(2\sqrt{x})$ , which suggests that  $2du = dx/\sqrt{x}$ . Now, re-examine our integral:

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = \int e^u 2du.$$

We can now evaluate this integral easily:

$$\int e^u 2du = 2 \int_e^u du = 2e^u + C = 2e^{\sqrt{x}} + C.$$