

Name: _____

Quiz Score: _____/10

Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

- 5 1. Using integration by parts, evaluate the following indefinite integral:

$$\int x \cos 5x \, dx$$

Solution: Here I spelled out that you should use integration by parts. You may not be so lucky on the final exam. Be able to recognize that this screams integration by parts because it is effectively two functions multiplied together, especially since one of which (x) is very easy to take a derivative of.

Using LIATE, we choose $u = x$ and $dv = \cos 5x \, dx$ which suggests that $du = dx$ and $v = \frac{1}{5} \sin 5x$. Plugging this into the integration by parts formula:

$$\begin{aligned} \int x \cos 5x \, dx &= \int u \, dv = uv - \int v \, du \\ &= \frac{1}{5} x \sin 5x - \int \frac{1}{5} \sin 5x \, dx \\ &= \frac{1}{5} x \sin 5x - \left[-\frac{1}{5} \cdot \frac{1}{5} \cos 5x \right] + C \\ &= \frac{1}{5} x \sin 5x + \frac{1}{25} \cos 5x + C \end{aligned}$$

- 5] 2. Using partial fraction decomposition, evaluate the following indefinite integral:

$$\int \frac{2x - 3}{x(x^2 + 1)} dx$$

Solution: Here we use a slightly nonstandard partial fraction decomposition. Notice $x^2 + 1$ is the case we discussed in class that is not factorable, thus our decomposition becomes:

$$\frac{2x - 3}{x(x^2 + 1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1}.$$

Finding a common denominator on the right side, and matching numerators we find:

$$2x - 3 = A(x^2 + 1) + (Bx + C)x = Ax^2 + A + Bx^2 + Cx.$$

We now consider the powers of x :

$$x^2 : 0 = A + B$$

$$x : 2 = C$$

$$\text{constant} : -3 = A.$$

From this, we can also conclude $A = -3$, $B = 3$, $C = 2$. Thus, we have now transformed our integral into:

$$\int \frac{2x - 3}{x(x^2 + 1)} dx = \int \left\{ -\frac{3}{x} + \frac{3x + 2}{x^2 + 1} \right\} dx = \int \left\{ -\frac{3}{x} + \frac{3x}{x^2 + 1} + \frac{2}{x^2 + 1} \right\} dx.$$

Each of these terms is easily to evaluate:

$$\int -\frac{3}{x} dx = -3 \ln|x|$$

$$\int \frac{3x}{x^2 + 1} dx = \frac{3}{2} \ln|x^2 + 1|, \quad \text{obtained by } u = x^2 + 1$$

$$\int \frac{2}{x^2 + 1} dx = 2 \arctan(x).$$

Thus, piecing this all together, we find:

$$\int \frac{2x - 3}{x(x^2 + 1)} dx = -3 \ln|x| + \frac{3}{2} \ln|x^2 + 1| + 2 \arctan(x) + C.$$