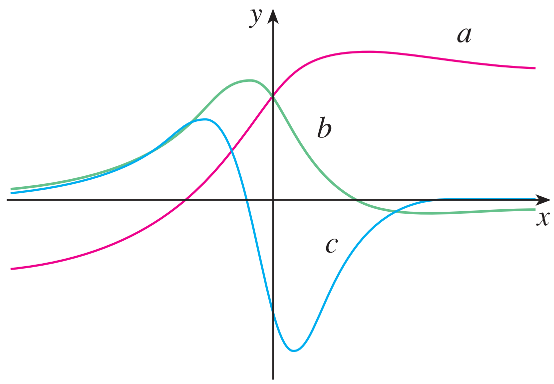
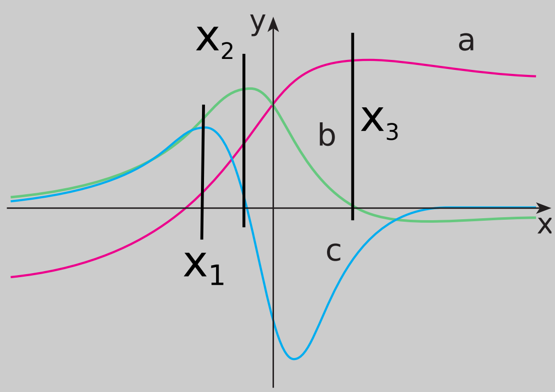
Name:____________________________________ Quiz Score:_____/10

Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

[4] 1. The graphs of $f(x), f'(x)$ and $f''(x)$ are shown below. Identify each curve and explain your choices.



Solution: Consider labeling the following points:



We can base our answer by referencing these points. Let us first examine x_2. We see that b is roughly flat around x_2, meaning its derivative must be 0. Immediately, this identifies c as a candidate. We can verify this by examining whether b is increasing or decreasing and how that relates to its derivative. Particularly, to the left of x_2, b is increasing, meaning its derivative must be positive. To the right of x_2, b is decreasing, meaning its derivative must be negative. The graph c does indeed satisfy this, meaning $b' = c$.

Next, let us examine c. We may think that around x_1, c is flat so its derivative must be 0, suggesting perhaps a as a candidate. Notice, though, to the left of x_1, c is increasing, but a is in fact, negative, eliminating a as a possible candidate for c', establishing $f'' = c$, by a process of elmination.

We can also see this by directly analyzing a. Notice around x_3, a is flat meaning its derivative must be 0, suggesting b as a candidate. To the left of x_3, a is always increasing, meaning its derivative must always be positive, which b is. To the right of x_3, a is decreasing slightly, which matches b going slightly negative. Thus, $a' = b$, soldifying our belief that $f = a, f' = b, f'' = c$.

2 2. True or false? If true, explain why. If it is false, provide a counterexample (a drawing suffices).

(a) If $f(x)$ is discontinuous at $x = a$, then $f(x)$ is not differentiable at $x = a$.

Solution: We have a theorem in class that says if a function is differentiable, it is continuous. In logical symbols, this means that D (differentiable) $\implies$ (implies) C (continuous). That is, if D is true, then C is true. The contrapositive says that if $D \implies C$, then $\{\text{not } C\} \implies \{\text{not } D\}$. That is, if a function is not continuous, it is not differentiable, which is exactly this statement, meaning it must be **true**.

Geometrically, if a function is discontinuous, what would it even mean to have a tangent line there? What point would the tangent line go through? It doesn't make sense to consider the tangent line to a discontinuous point, so this further solidfies our answer.

(b) If a function $f(x)$ is continuous at $x = a$, then $f(x)$ is always differentiable at $x = a$.

Solution: We have seen in class if we have a corner or a vertical slope, $f'(x)$ does not exist, but neither of these conditions violates continuity, providing counterexamples to the statement. Thus, the statement is, in general, **false**.

4 3. Compute the derivative of the following function **using the limit definition of the derivative**:

$f(x) = x^2 + x.$

Solution: We know the definition of $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$. Applying this:

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{(x+h)^2+(x+h)-x^2-x}{h} \\
&= \lim_{h\to 0} \frac{x^2+2xh+h^2+x+h-x^2-x}{h} \\
&= \lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2+\cancel{x}+h-\cancel{x^2}-\cancel{x}}{h} \\
&= \lim_{h\to 0} \frac{h(2x+h+1)}{h} \\
&= \lim_{h\to 0} \frac{\cancel{h}(2x+h+1)}{\cancel{h}} \\
&= \lim_{h\to 0} 2x+h+1 \\
&= 2x+1.
\end{aligned}
$$

Note, we can easily verify this with the power rule that we recently learned, which states $\frac{d}{dx}(x^n) = nx^{n-1}$, which immediately provides us the same answer.