Name:________________________________________ Quiz Score:_____/10

Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

[4] 1. Find the equation to the tangent line to the curve at the specified point:

$$y = \frac{2^x}{\cos x}, \qquad (0, 1)$$

**Solution:** To compute the tangent line, we need the derivative $\mathrm{d}y/\mathrm{d}x$. There are two ways, either quotient rule or product rule. First, we will attempt the quotient rule method by letting $f(x) = 2^x$ and $g(x) = \cos x$. Then, we know:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2}.$$

We know from class that:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(a^x\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{\ln a x}\right) = e^{\ln a x}\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln a x\right) = \ln(a)\, a^x$$

Thus, we can conclude $f'(x) = \ln(2)2^x$. We also know from class that $g'(x) = -\sin(x)$, thus piecing this all together:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2} = \frac{\ln(2)2^x \cos x - (-\sin x)2^x}{(\cos x)^2} = \frac{2^x\left[\ln(2)\cos x + \sin x\right]}{\cos^2 x}.$$

Evaluating at $x = 0$ is nontrivial, but doable. Notice we can split this sum:

$$\frac{2^x\left[\ln(2)\cos x + \sin x\right]}{\cos^2 x} = 2^x\left[\ln(2)\frac{1}{\cos x} + \frac{\tan x}{\cos x}\right]$$

Now, consider $\lim x \to 0$, in which case $2^x \to 1$, meaning it does not matter. We also know that $\tan x \to 0$ as $x \to 0$ and $\cos x \to 1$ as $x \to 0$, meaning the second term goes to 0 and the first term goes to $\ln(2)$. Thus, we're left with just $\ln(2)$. Thus, $f'(0) = \ln 2$, meaning our tangent line is $y - 1 = \ln(2)(x - 0) \implies y = \ln(2)x + 1$.

Note, we could have obtained this also with product rule by rewriting $y = 2^x \sec x$ and recalling that $\mathrm{d}/\mathrm{d}x \sec = \tan x \sec x$, providing us with the same derivative.

2. Compute the derivative of each of the following functions:

3 (a) $f(x) = \sqrt[3]{x^6 + 3x^5 + 4x - 2}$.

**Solution:** Recall our rule for derivatives of this form:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{[\phi(x)]^n\right\} = n[\phi(x)]^{n-1}\phi'(x).$$

Here, $\phi(x) = x^6 + 3x^5 + 4x - 2$ and $n = 1/3$. By the power rule, $\phi'(x) = 6x^5 + 15x^4 + 4$, meaning we can plug this into our rule:

$$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left\{[\phi(x)]^n\right\} = n[\phi(x)]^{n-1}\phi'(x) = \frac{1}{3}\left[x^6 + 3x^5 + 4x - 2\right]^{-2/3}\left(6x^5 + 15x^4 + 4\right).$$

3 (b) $g(x) = e^{x \sin x}$.

**Solution:** Recall our solution to derivatives of the following form:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{e^{u(x)}\right\} = e^{u(x)}u'(x).$$

Here, our $u(x) = x \sin x$, which by product rule is $u'(x) = 1 \cdot \sin x + x \cos x$. Thus, we can evaluate our derivative using the above rule:

$$g'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left\{e^{u(x)}\right\} = e^{u(x)}u'(x) = e^{x \sin x}\left(\sin x + x \cos x\right).$$