34. **(a)** If the three planes are parallel and distinct, then they have no common point of intersection, so the system has no solution.

(b) If the three planes coincide, then each of the infinitely many different points (x, y, z) of this common plane provides a solution of the system.

(c) If two of the planes coincide and are parallel to the third plane, then the three planes have no common point of intersection, so the system has no solution.

(d) If two of the planes intersect in a line that is parallel to the third plane, then the three planes have no common point of intersection, so the system has no solution.

(e) If two of the planes intersect in a line that lies in the third plane, then each of the infinitely many different points (x, y, z) of this line provides a solution of the system.

(f) If two of the planes intersect in a line that intersects the third plane in a single point, then this point (x, y, z) provides the unique solution of the system.

SECTION 3.2

MATRICES AND GAUSSIAN ELIMINATION

Because the linear systems in Problems 1–10 are already in echelon form, we need only start at the end of the list of unknowns and work backwards.

1. Starting with $x_3 = 2$ from the third equation, the second equation gives $x_2 = 0$, and then the first equation gives $x_1 = 1$.

2. Starting with $x_3 = -3$ from the third equation, the second equation gives $x_2 = 1$, and then the first equation gives $x_1 = 5$.

3. If we set $x_3 = t$ then the second equation gives $x_2 = 2 + 5t$, and next the first equation gives $x_1 = 13 + 11t$.

4. If we set $x_3 = t$ then the second equation gives $x_2 = 5 + 7t$, and next the first equation gives $x_1 = 35 + 33t$.

5. If we set $x_4 = t$ then the third equation gives $x_3 = 5 + 3t$, next the second equation gives $x_2 = 6 + t$, and finally the first equation gives $x_1 = 13 + 4t$.

6. If we set $x_3 = t$ and $x_4 = -4$ from the third equation, then the second equation gives $x_2 = 11 + 3t$, and next the first equation gives $x_1 = 17 + t$.

7. If we set $x_3 = s$ and $x_4 = t$, then the second equation gives $x_2 = 7 + 2s - 7t$, and next the first equation gives $x_1 = 3 - 8s + 19t$.

8. If we set $x_2 = s$ and $x_4 = t$, then the second equation gives $x_3 = 10 - 3t$, and next the first equation gives $x_1 = -25 + 10s + 22t$.

9. Starting with $x_4 = 6$ from the fourth equation, the third equation gives $x_3 = -5$, next the second equation gives $x_2 = 3$, and finally the first equation gives $x_1 = 1$.

10. If we set $x_3 = s$ and $x_5 = t$, then the third equation gives $x_4 = 5t$, next the second equation gives $x_2 = 13s - 8t$, and finally the first equation gives $x_1 = 63s - 16t$.

In each of Problems 11–22, we give just the first two or three steps in the reduction. Then we display a resulting echelon form **E** of the augmented coefficient matrix **A** of the given linear system, and finally list the resulting solution (if any). The student should understand that the echelon matrix **E** is not unique, so a different sequence of elementary row operations may produce a different echelon matrix.

11. Begin by interchanging rows 1 and 2 of **A**. Then subtract twice row 1 both from row 2 and from row 3.

$$\mathbf{E} = \begin{bmatrix} 1 & 3 & 2 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \end{bmatrix}; \quad x_1 = 3,\ x_2 = -2,\ x_3 = 4$$

12. Begin by subtracting row 2 of **A** from row 1. Then subtract twice row 1 both from row 2 and from row 3.

$$\mathbf{E} = \begin{bmatrix} 1 & -6 & -4 & 15 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix}; \quad x_1 = 5,\ x_2 = -3,\ x_3 = 2$$

13. Begin by subtracting twice row 1 of **A** both from row 2 and from row 3. Then add row 2 to row 3.

$$\mathbf{E} = \begin{bmatrix} 1 & 3 & 3 & 13 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \quad x_1 = 4 + 3t,\ x_2 = 3 - 2t,\ x_3 = t$$

14. Begin by interchanging rows 1 and 3 of **A**. Then subtract twice row 1 from row 2, and three times row 1 from row 3.

2. $\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \xrightarrow{R1-R2} \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} \xrightarrow{R2-2R1} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \xrightarrow{R1-2R2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3. $\begin{bmatrix} 3 & 7 & 15 \\ 2 & 5 & 11 \end{bmatrix} \xrightarrow{R1-R2} \begin{bmatrix} 1 & 2 & 4 \\ 2 & 5 & 11 \end{bmatrix} \xrightarrow{R2-2R1} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 3 \end{bmatrix} \xrightarrow{R1-2R2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix}$

4. $\begin{bmatrix} 3 & 7 & -1 \\ 5 & 2 & 8 \end{bmatrix} \xrightarrow{R2-R1} \begin{bmatrix} 3 & 7 & -1 \\ 2 & -5 & 9 \end{bmatrix} \xrightarrow{R1-R2} \begin{bmatrix} 1 & 12 & -10 \\ 2 & -5 & 9 \end{bmatrix}$

$\xrightarrow{R2-2R1} \begin{bmatrix} 1 & 12 & -10 \\ 0 & -29 & 29 \end{bmatrix} \xrightarrow{(-1/29)R2} \begin{bmatrix} 1 & 12 & -10 \\ 0 & 1 & -1 \end{bmatrix} \xrightarrow{R1-12R2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}$

5. $\begin{bmatrix} 1 & 2 & -11 \\ 2 & 3 & -19 \end{bmatrix} \xrightarrow{R2-2R1} \begin{bmatrix} 1 & 2 & -11 \\ 0 & -1 & 3 \end{bmatrix} \xrightarrow{(-1)R2} \begin{bmatrix} 1 & 2 & -11 \\ 0 & 1 & -3 \end{bmatrix} \xrightarrow{R1-2R2} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -3 \end{bmatrix}$

6. $\begin{bmatrix} 1 & -2 & 19 \\ 4 & -7 & 70 \end{bmatrix} \xrightarrow{R2-4R1} \begin{bmatrix} 1 & -2 & 19 \\ 0 & 1 & -6 \end{bmatrix} \xrightarrow{R1+2R2} \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & -6 \end{bmatrix}$

7. $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 1 \\ 2 & 1 & 9 \end{bmatrix} \xrightarrow{R2-R1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -2 \\ 2 & 1 & 9 \end{bmatrix} \xrightarrow{R3-2R1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -2 \\ 0 & -3 & 3 \end{bmatrix}$

$\xrightarrow{(1/2)R2} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 0 & -3 & 3 \end{bmatrix} \xrightarrow{R3+3R2} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R1-2R2} \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

8. $\begin{bmatrix} 1 & -4 & -5 \\ 3 & -9 & 3 \\ 1 & -2 & 3 \end{bmatrix} \xrightarrow{R2-3R1} \begin{bmatrix} 1 & -4 & -5 \\ 0 & 3 & 18 \\ 1 & -2 & 3 \end{bmatrix} \xrightarrow{R3-R1} \begin{bmatrix} 1 & -4 & -5 \\ 0 & 3 & 18 \\ 0 & 2 & 8 \end{bmatrix}$

$\xrightarrow{R2-R3} \begin{bmatrix} 1 & -4 & -5 \\ 0 & 1 & 10 \\ 0 & 2 & 8 \end{bmatrix} \xrightarrow{R3-2R2} \begin{bmatrix} 1 & -4 & -5 \\ 0 & 1 & 10 \\ 0 & 0 & -12 \end{bmatrix} \xrightarrow{(-1/12)R3} \cdots \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$