

Name: _____

Quiz Score: _____/10

Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

- 5 1. Find an explicit solution for the following differential equation for $y(x)$:

$$\frac{dy}{dx} = \pi\sqrt{xy}.$$

Solution: The intent of this problem was that you recognize immediately that it is **separable**. Thus, we rewrite it in the suggestive form

$$\frac{dy}{\sqrt{y}} = \pi\sqrt{x}dx.$$

From here, we can just integrate both sides, on the left: with respect to y and on the right: with respect to x , yielding

$$\int \frac{dy}{\sqrt{y}} = \int \pi\sqrt{x}dx.$$
$$2\sqrt{y} = \frac{2\pi}{3}x^{3/2} + C,$$

which we can solve explicitly for y to yield

$$y = \left(\frac{\pi}{3}x^{3/2} + C\right)^2.$$

Note, we don't have an initial condition, so we're left with a family of solutions described by some parameter C .

- 5 2. Find an explicit solution for the following differential equation for $y(x)$:

$$xy' - 3y = x^3.$$

Solution: Unlike the first problem, this is not separable, but rather a first order, linear differential equation. Thus, we are going to use an integrating factor. Our general form of which is

$$y'(x) + P(x)y = Q(x).$$

When we rewrite our equation in this form, we see

$$y'(x) - \frac{3}{x}y = x^2 \quad \implies \quad P(x) = -\frac{3}{x}, \quad Q(x) = x^2.$$

Our integrating factor is then

$$\mu(x) = e^{\int P(x) dx} = e^{\int -3/x dx} = e^{-3 \ln x} = x^{-3}.$$

We know that for a general first order, linear ODE, after multiplying both sides of the equation our integrating factor, we're left with

$$\frac{d}{dx} \{ \mu(x)y(x) \} = \mu(x)Q(x).$$

In our particular case, that is

$$\frac{d}{dx} \{ x^{-3}y(x) \} = \frac{1}{x}.$$

When we integrate both sides, this yields

$$x^{-3}y(x) = \ln x + C,$$

which then suggests that, solving for $y(x)$

$$y(x) = x^3 \ln x + Cx^3.$$

Again, due to the lack of an initial condition, we have no way of solving for C , but this is perfectly fine.