

Name: _____

Quiz Score: _____/10

Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

- 8 1. Consider the initial problem

$$\frac{dx}{dt} = t^2 - x, \quad x(0) = 1.$$

Construct an approximation to the solution $x(t)$ on the interval $[0, 1]$ using Euler's method with step-size $h = 0.5$.

Solution: Note that what we normally call x, y here, we actually have t, x . It doesn't matter what we call them, but it's important to keep track of what the independent and dependent variables are for our differential equation. Euler's method, in algorithm says:

$$x_{n+1} = x_n + hk, \quad k := f(t_n, x_n).$$

Thus, in this problem, we have $h = 1/2$, meaning we have 3 t_j values: $t_0 = 0, t_1 = 0.5, t_2 = 1$. We know the initial value $x_0 = 1$. Thus, at the first step:

$$x_1 = x_0 + hf(t_0, x_0) = 1 + 0.5 [0^2 - 1] = 0.5.$$

We perform one more step:

$$x_2 = x_1 + hf(t_1, x_1) = 0.5 + 0.5 [0.5^2 - 0.5] = 0.375.$$

Thus, the values x_0, x_1, x_2 approximate the true solution values $x(0), x(0.5), x(1)$.

- 2 2. If we instead took a step-size of $h = 0.05$, what can we say about the change in the maximum possible error? Make sure to clearly state any theorems you refer to.

Solution: We have a theorem that says there exists some constant C such that the cumulative error up until some step n , described below, satisfies:

$$|\text{error}| = |x(t_n) - x_n| \leq Ch.$$

In other words, we know how the maximum cumulative error scales as we change h . Specifically, if we reduce h by a factor of $1/10$ as we are hypothesizing, we can say that the maximum possible error is reduced by the same factor.