HW7 solutions 9.1) 14, 30 9.3) 8, 14, 30
9.2) 16, 20 9.4) 6, 10, 24

9.1.14) $x^2+y^2+z^2+8x-6y+2z+17=0$.

Must complete the square in x, y, z:

$x^2+8x = (x+4)^2-16$.

$y^2-6y = (y-3)^2-9$

$z^2+2z = (z+1)^2-1$.

Thus:

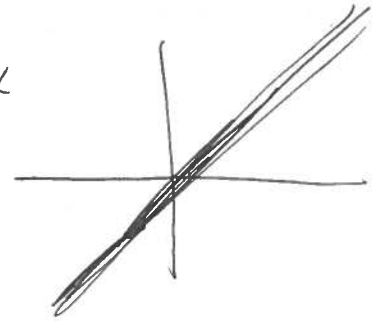
$(x+4)^2+(y-3)^2+(z+1)^2-9=0$.

$\boxed{\text{Thus, center @ } (-4,3,-1). \quad r=3.}$

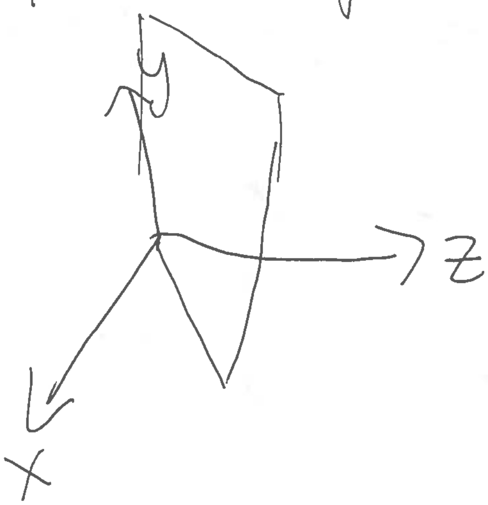
9.1.30) $x = z$.

Think about 2D:

$y = x$



Same idea. We get a plane:



9.2.16) $\vec{a} = 4\vec{i} + \vec{j}$, $\vec{b} = \vec{i} - 2\vec{j}$.

$\vec{a} + \vec{b} = 5\vec{i} - \vec{j}$.

$2\vec{a} + 3\vec{b} = 11\vec{i} - 4\vec{j}$.

$|\vec{a}| = \sqrt{(4)^2 + (1)^2} = \sqrt{17}$.

$|\vec{a} - \vec{b}| = |3\vec{i} + 3\vec{j}|$

$= \sqrt{(3)^2 + (3)^2} = 3\sqrt{2}$.

9.2.20) $\vec{a} = \langle -4, 2, 4 \rangle$.

Unit vector:

$\frac{\vec{a}}{|\vec{a}|} =$

$|\vec{a}| = \sqrt{(4)^2 + (2)^2 + (4)^2}$

$= 6$.

$\frac{\langle -4, 2, 4 \rangle}{6} = \langle -\frac{2}{3}, \frac{1}{3}, \frac{2}{3} \rangle$

magnitude = 1.

9.3.8) $\vec{a} = \langle p, -p, 2p \rangle$

$\vec{b} = \langle 2q, q, -q \rangle$.

$a \cdot b =$

$p \cdot (2q) + (-p) \cdot q + (2p)(-q)$

$\boxed{=-pq}$.

9.3.14) a hamburgers @ ~~\$2~~ \$2.00 each

b hot dogs, @ \$1.50 each

c soft drinks. @ \$1.00 each

Thus,

$$A \cdot P = 2a + 1.5b + 1.c = \text{total cost.}$$

9.3.30) $\vec{a} = \langle 1, 2 \rangle$, $\vec{b} = \langle -4, 1 \rangle$

scalar proj: $\text{comp}_a b = \dfrac{\vec{a} \cdot \vec{b}}{\vec{a}}$

$$|\vec{a}| = \sqrt{1^2 + 2^2} = \sqrt{5}.$$

$$a \cdot b = (1)(-4) + (2)(1) = -2.$$

Thus, $\text{comp}_a b = \dfrac{-2}{\sqrt{5}}.$

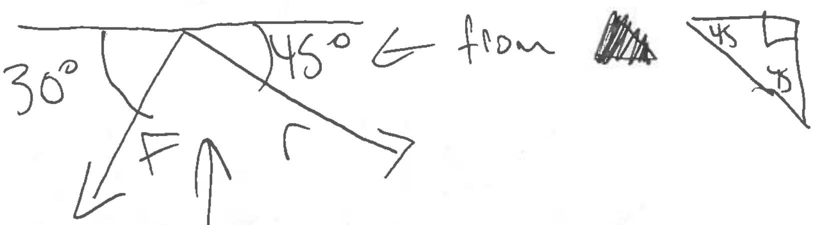
Then we know

$$\text{proj}_a b = \text{comp}_a b \cdot \frac{a}{|a|}$$

$$= \frac{-2}{\sqrt{5}} \cdot \frac{\langle 1,2 \rangle}{\sqrt{5}} = \boxed{\left\langle -\frac{2}{5}, -\frac{4}{5} \right\rangle}.$$

9, 4, 6)

recall tail to tail hint in class!



must be $\theta = 105°$.

We know $\vec{\tau} = \vec{r} \times \vec{F}$, we want just magnitude so:

$$|\vec{\tau}| = |\vec{r} \times \vec{F}|.$$

Using our x-prod definition:

$$|\vec{\tau}| = |\vec{r} \times \vec{F}| = |\vec{r}||\vec{F}| \cdot \sin 105°$$

$$= 4\sqrt{2} \cdot 36 \cdot \frac{(1+\sqrt{3})}{(2\sqrt{2})} \approx \boxed{196.7 \text{ ft} \cdot \text{lbs}}$$

9.4.10) $\vec{a} = 0\vec{i} + \vec{j} + 7\vec{k}$
$\vec{b} = 2\vec{i} - \vec{j} + 4\vec{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 7 \\ 2 & -1 & 4 \end{vmatrix}$$

doing math while sick is hard

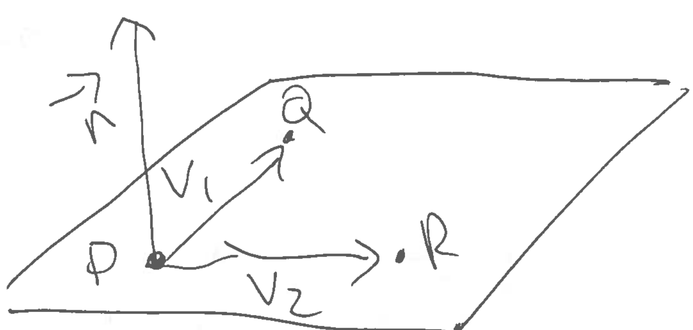
$=$

$$\vec{i}\begin{vmatrix}1 & 7\\ -1 & 4\end{vmatrix} - \vec{j}\begin{vmatrix}0 & 7\\ 2 & 4\end{vmatrix} + \vec{k}\begin{vmatrix}0 & 1\\ 2 & -1\end{vmatrix}$$

$$= \vec{i}\left(1(4)-(-1)7\right) - \vec{j}\left(0(4)-7\cdot 2\right) + \vec{k}\left(0(-1)-2\cdot 1\right)$$

$$= 11\vec{i} + 14\vec{j} - 2\vec{k}.$$

a.4.24)



$P = (-1,3,1)$ $Q = (0,5,2)$

$R = (4,3,-1).$

$\vec{v}_1 = \overrightarrow{PQ} = (1,2,1)$

$\vec{v}_2 = \overrightarrow{PR} = (5,0,-2).$

$$\vec{n} = \vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ 5 & 0 & -2 \end{vmatrix}$$

$$= \langle -4, 7, -10 \rangle.$$

Recall:

Area of trapezoid $= |\vec{v}_1 \times \vec{v}_2|$.

Thus, of $\Delta = \frac{1}{2}$ (Area of trapezoid).

Therefore:

Area of triangle $= \frac{1}{2}|\vec{v}_1 \times \vec{v}_2|$

$$= \frac{1}{2}\left(\sqrt{165}\right)$$

$$\approx 12.845.$$