HW8 Solutions

9.5) 4, 18, 24, 58
9.6) 14, 22
9.7) 4, 6, 16, 28
10.1) 4, 8, (19-24)

9.5.4) $P = (0, 14, -10)$

L_2: $x = -1+2t$
$y = 6-3t$
$z = 3+9t$

so $\vec{v}_2 = \langle 2, -3, 9 \rangle$.

Thus:

$L = \vec{r_0} + \vec{v}t = \langle 0+2t, 14+(-3)t, -10+9t \rangle$

9.5.18)

L_1: $x = 1+2t$, $y = 3t$, $z = 2-t$

L_2: $x = -1+s$, $y = 4+s$, $z = 1+3s$.

x: $1+2t = -1+s$
y: $3t = 4+s$

$\rightarrow$ $s = 14$, $t = 6$.

Check z:

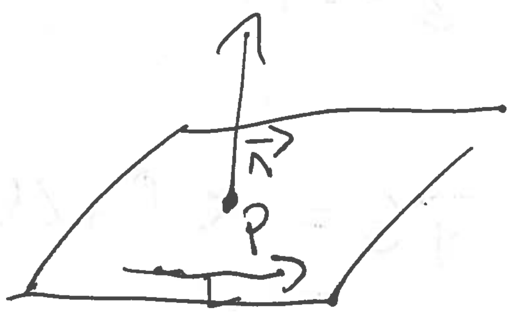
$z_1 = 2 - 6 = -4$

$z_2 = 1 + 3s = 1 + 3(14) = 43$

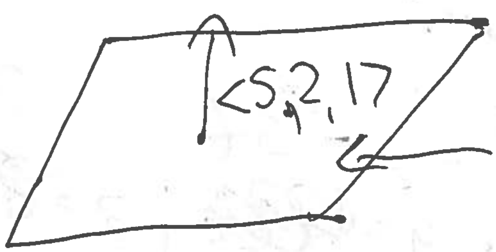
$-4 \neq 43$, so

no intersection.

9.5.24)



We need a point and the normal vector.



$5x + 2y + z = 1$

so $\vec{n} = \langle 5, 2, 1 \rangle$,

How do we find a point? Any point on the line.

Say $t = 0$:

$P = (1, 2, 4)$, so:

Plane:

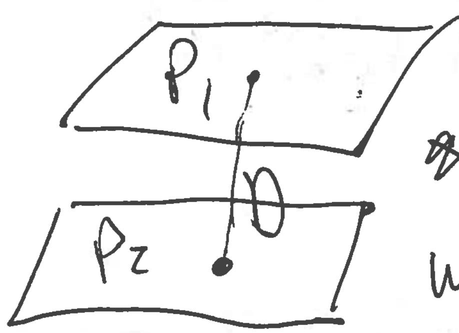
$$\langle 5,2,1\rangle \cdot \langle x-1, y-2, z-4\rangle = 0.$$

$$5(x-1)+2(y-2)+(z-4)=0.$$

9.5.58) P_1: $6z = 4y - 2x \Rightarrow 2x-4y+6z=0$

P_2: $9z = 1-3x+6y \Rightarrow 3x-6y+9z=1$

parallel.



We can choose any point on P_1, say ~~pt~~ $(0,0,0)$.

Now, we just have distance from pt to plane:

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}}.$$

where here,

$$\langle a,b,c\rangle = \vec{n}_2 = \langle 3,-6,9\rangle$$

and $d=1$.

$(x_0,y_0,z_0)=(0,0,0)$,

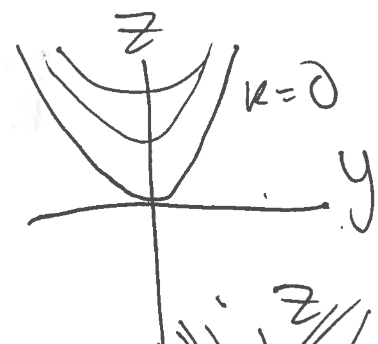
so:

$$D = \frac{|1|}{\sqrt{3^2+(-6)^2+9^2}} = \frac{1}{3\sqrt{14}}.$$

9.6.14) $f(x,y)=x^2+y^2=z$

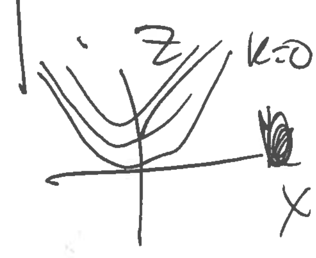
if $x=k$:

$$z=k^2+y^2$$

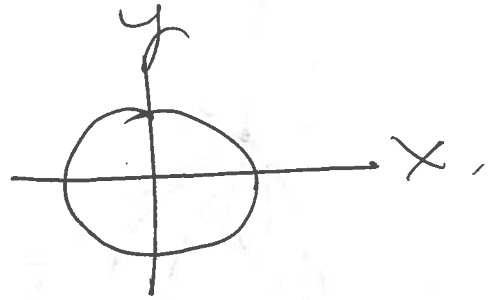


if $y=k$

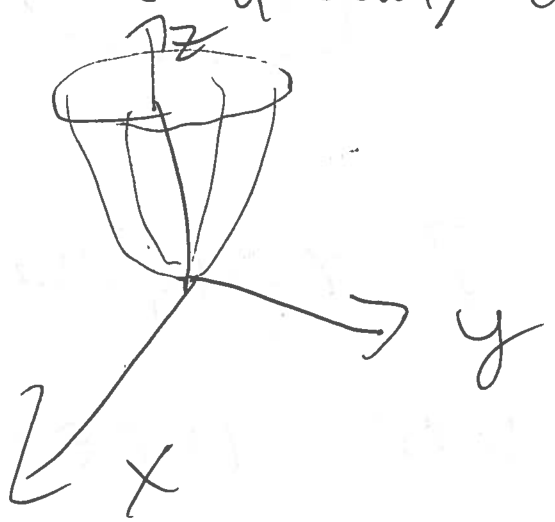
$$z=x^2+k^2$$



$z=k \Rightarrow \quad k=x^2+y^2$

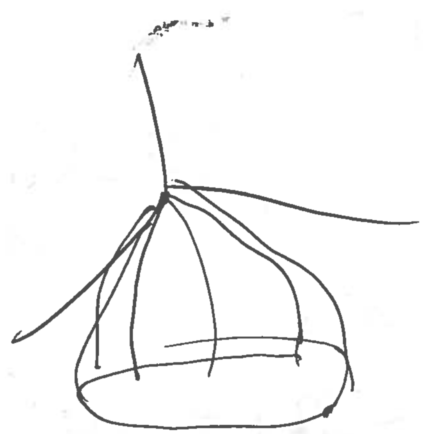


so we have a bowl/ elliptic paraboloid,



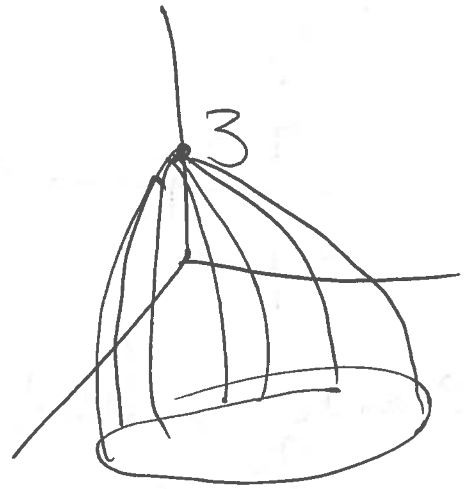
b.) $z=-x^2-y^2 = -(x^2+y^2)$

so this just flips it



c. $z=3-x^2-y^2$ is just 3 + previous picture

So:



9.6.22) $4y^2+z^2-x-16y-4z+20=0$

We must complete the square in x, y, z.

for example:

$4y^2-16y = 4(y^2-4y)$.

and

$y^2-4y = (y-2)^2-4$ know how to do this so:

$$4(y^2 - 4y) = 4((y-2)^2 - 4).$$

~~[illegible]~~ similarly,

$$z^2 - 4z = (z-2)^2 - 4.$$

so we have:

~~[illegible]~~

$$4((y-2)^2 - 4) + (z-2)^2 - 4 - x + 20 = 0$$

$$4(y-2)^2 - 16 + (z-2)^2 - 4 - x + 20 = 0$$

Cancel

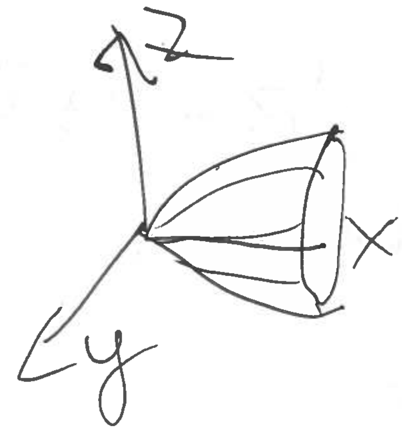
$$4(y-2)^2 + (z-2)^2 = x.$$

shifts by 2 in z, y. Doesnt affect shape,

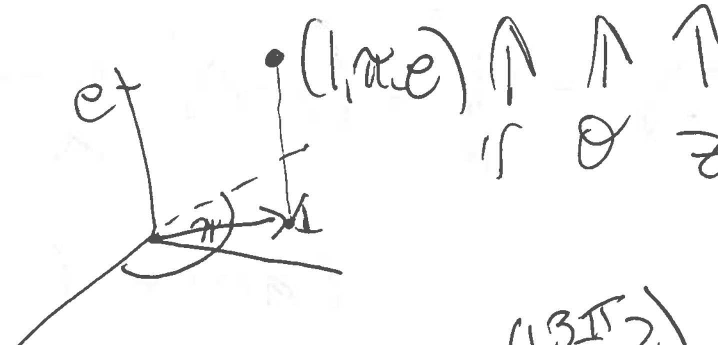
We roughly have:

$\frac{y^2}{a} + \frac{z^2}{b} = \frac{x}{c}$ In table, this is

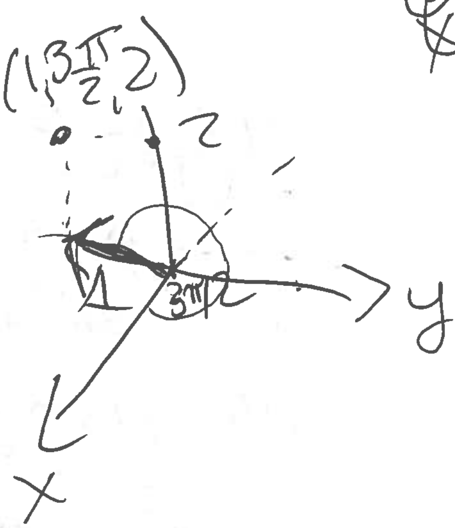
An elliptic paraboloid or bowl opening in x-direction:



a.7.4) a.) $(1, \pi, e)$



b.) $(1, 3\pi/2, 2)$



9.7.6) $(2\sqrt{3}, 2, -1)$

$\quad\quad x \quad y \quad z$

$r = x^2 + y^2 = 4$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = 30^\circ$

$z = -1.$

$(4, -3, 2)$

$r = x^2 + y^2 = 5$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(-\frac{3}{4}\right)$

$z = -2$

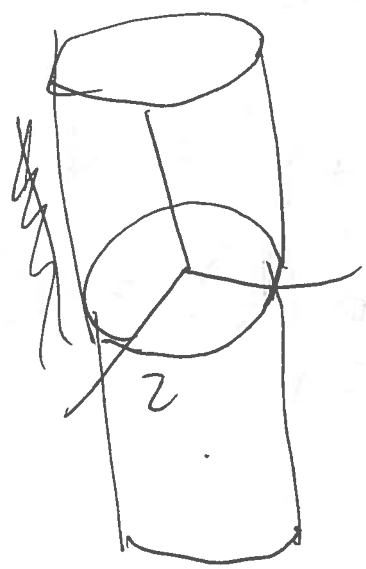
$\approx 360^\circ - 36^\circ.$

9.7.16) $\rho \sin\phi = 2$

but recall $\rho\sin\phi = r$. in cylindrical.

so we have $r = 2$, in cylindrical

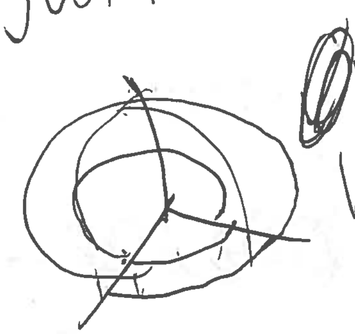
which is just a cylinder!



9.7.28) $2 \le \rho \le 3$, $\pi/2 \le \phi \le \pi$

θ is free, so we get full rotation around z-axis.

just this!



hollow sphere.

only top half of sphere

So, combining, we get top half of hollow sphere.

10.1 4) $\lim_{t\to 0} \langle \frac{e^t-1}{t}, \frac{\sqrt{1+t}-1}{t}, \frac{3}{1+t} \rangle$

Termwise limits:

$$\lim_{t\to 0} \frac{e^t-1}{t} = \lim_{t\to 0} \frac{e^t}{1} \text{ by L'Hopital}$$

$$= 1.$$

$\lim_{t\to 0} \frac{\sqrt{1+t}-1}{t}$ again by L'Hopital: $\lim_{t\to 0} \frac{(1+t)^{-3/2}\cdot\frac{1}{2}}{1}$

$= \frac{1}{2}$.

Lastly, $\lim_{t \to 0} \frac{3}{1+t} = 0$.

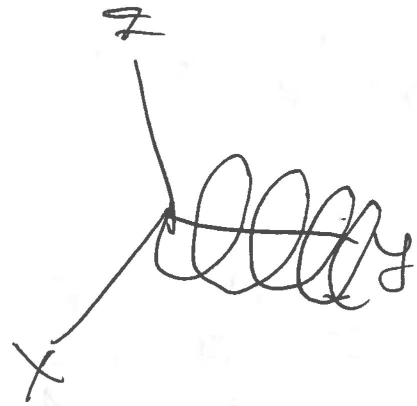
So, $\lim_{t \to 0} = \langle 1, \frac{1}{2}, 0 \rangle$.

10.1.8) $\vec{r} = \langle \sin \pi t, t, \cos \pi t \rangle$

grows

Circle in x, z. Note π is just "speed".

so, helix growing in y.



10.1.19-24)

19 – II – notice it's a helix with increasing radius

20 – VI – height → 0, as t → 0, helix.

21 – V – grows in x, z without bound

22 – I – stays bounded in x, y, z,

23 – IV – helix, height grows slowly

24 – III – grows linearly in z, wobbles in x, y.