1320 HW9

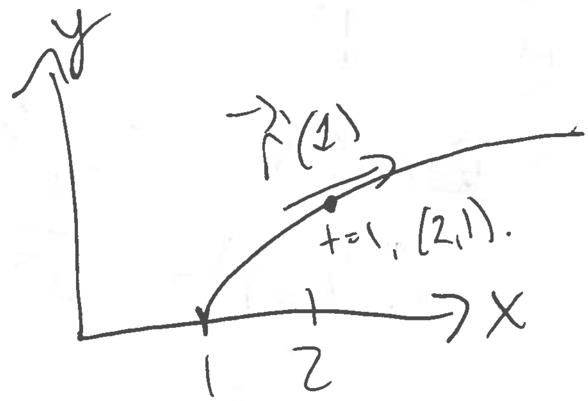
10.2) 4, 10, 16, 24, 36

10.3) 2, 18, 22, 36, 58.

10.2.4) $\vec{r} = \langle 1+t, \sqrt{t} \rangle$, $t = 1$.

Notice $x = 1+t$ so $t = x-1$.
$y = \sqrt{t}$

$y = \sqrt{x-1}$.



$\vec{r}'(t) = \langle 1, \frac{1}{2}t \rangle$.

$\vec{r}'(1) = \langle 1, \frac{1}{2} \rangle$.

10.2.10) $\vec{r}(t) = \langle \tan t, \sec t, \frac{1}{t^2} \rangle$.

$\vec{r}'(t) = \langle \sec^2 t, \sec t \tan t, -\frac{2}{t^3} \rangle$.

10.2.16) $\vec{r} = \langle 4\sqrt{t}, t^2, t \rangle$.

$\vec{r}'(t) = \langle \frac{2}{\sqrt{t}}, 2t, 1 \rangle$.

$\vec{T}(t) = \dfrac{r'(t)}{|r'(t)|}$.

@ $t = 1$:

$r'(1) = \langle 2, 2, 1 \rangle$.

$|r'(1)| = \sqrt{2^2 + 2^2 + 1} = 3$,

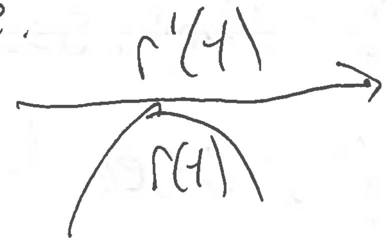
so:

$T(1) = \langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle$,

10.2.24) $x = \ln t$, $y = 2\sqrt{t}$, $z = t^2$ @ $(0, 2, 1)$

$t = 1$.

Tangent line:



so $\vec{r} = \langle \ln t, 2\sqrt{t}, t^2 \rangle$

$\vec{r}'(t) = \langle \frac{1}{t}, \frac{1}{\sqrt{t}}, 2t \rangle$.

$\vec{r}'(1) = \langle 1, 1, 2 \rangle$.

so

Line: $\vec{r} = \vec{r_0} + \vec{v}t$.

direction $= \vec{r}'(1)$

$= \langle 0, 2, 1 \rangle + \langle 1, 1, 2 \rangle t$.

10.2.36) $\int_1^2 \langle t^2, t\sqrt{t-1}, t\sin \pi t \rangle$.

$= \langle \frac{t^3}{3}, \frac{2}{15}(t-1)^{3/2}(3t+2), \frac{\sin \pi t - \pi t \cos \pi t}{\pi^2} \rangle$

eval @ $t=2$ $-$ $t=1$

yields:

$$\int_1^2 \vec{r}\,dt \quad \left\langle \frac{7}{3}, \frac{16}{15}, -\frac{3}{\pi} \right\rangle,$$

10.3.2) $\vec{r} = \left\langle 2t, t^2, \frac{t^3}{3} \right\rangle,\ 0 \le t \le 1$

$$L = \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt.$$

$$= \int_0^1 \sqrt{(2)^2 + (2t)^2 + (t^2)^2}\,dt$$

$$= \int \sqrt{4 + 4t^2 + t^4}\,dt$$

$$= \int \sqrt{(2+t)^2}\,dt$$

$$= \int (2+t)\,dt$$

$$= 2t + \frac{t^2}{2} \Big|_{t=0}^{t=1} = \frac{5}{2}.$$

10.3.18) $\vec{r}(t) = \langle t^2, \sin t - t\cos t, \cos t + t\sin t \rangle.$

$$T = \frac{r'}{|r'|}. \quad N = \frac{T'}{|T'|}.$$

so

$$r'(t) = \langle 2t, \cancel{\cos t} - \cancel{\cos t} + t\sin t, t\cos t \rangle.$$

$$|r'| = \sqrt{(2t)^2 + (t\sin t)^2 + (t\cos t)^2}$$

$$= \sqrt{4t^2 + t^2(1)}$$

$$= \sqrt{5t^2}$$

$$= t\sqrt{5},$$

so:

$$T = \frac{1}{\sqrt{5}t} \langle 2t, t\sin t, t\cos t \rangle.$$

We now have

$$T' = \left\langle 0, \frac{1}{\sqrt{5}}\cos t, -\frac{1}{\sqrt{5}}\sin t \right\rangle$$

so $|T'| = \dfrac{1}{\sqrt{5}}$,

and

$$N = \frac{T'}{|T'|} = \langle 0, \cos t, -\sin t \rangle.$$

Curvature $\kappa(t) = \dfrac{|T'(t)|}{|r'(t)|} = \dfrac{1/\sqrt{5}}{\sqrt{5}\,t} = \dfrac{1}{5t}$.

usually don't use this formula but easier since we have T, r'.

10.3.22) $r(t) = \langle t, t^2, e^t \rangle$.

$$k = \frac{|r' \times r''|}{|r'|^3}$$

$$r' = \langle 1, 2t, e^t \rangle$$

$$r'' = \langle 0, 2, e^t \rangle$$

$$|r'| = \sqrt{1^2 + 4t^2 + e^{2t}}.$$

$$r' \times r'' = \begin{vmatrix} i & j & k \\ 1 & 2t & e^t \\ 0 & 2 & e^t \end{vmatrix}$$

$$= \langle -2e^t + 2e^t t, -e^t, 2 \rangle.$$

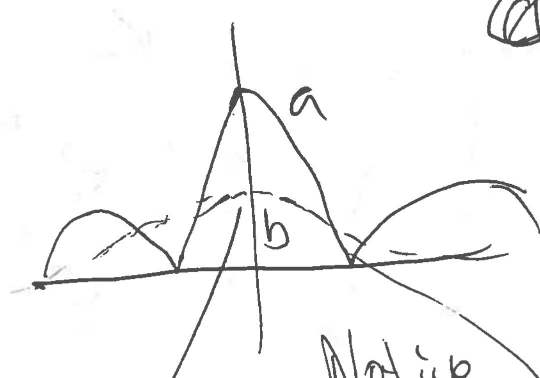
so $|r' \times r''| = \sqrt{4 + e^{2t} + (-2e^{t} + 2e^{t}t)^2}$.

so

$k =$ some mess

$$= \frac{\sqrt{4 + e^{2t} + (-2e^{t} + 2e^{t}t)^2}}{\sqrt{1 + 4t^2 + e^{2t}}}.$$

10.3.36)



Notice b is flat here or straight.

curvy b, hinh a and $a = 0$.

so $b = f(x)$, $a = k$.

10.3.58)

Want to prove:

$$\frac{dN}{ds} = -\kappa T + \tau B.$$

so:

$$N = B \times T.$$

product rule:

$$\frac{dN}{ds} = \frac{dB}{ds} \times T + B \times \frac{dT}{dS}.$$

from formulae 1, 3:

$$\frac{dT}{ds} = \kappa N$$

$$\frac{dB}{ds} = -\tau N, \text{ so:}$$

$$\frac{dN}{ds} = -\tau N \times T + B \times \kappa N.$$

$$= -\tau(N \times T) + \kappa(B \times N).$$

$$= \tau(T \times N) - \kappa(N \times B).$$

$$= \tau(B) - \kappa(T).$$

$$= \tau B - \kappa T$$

what we wanted to prove.